表5-2　ベクトルの回転

| No. | 方向余弦マトリクスによる回転 | | | | 固定角型回転マトリクスによる回転 | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | $\mathbf{a}^{I}_{L} = \mathbf{D}^{I}_{L} \cdot \mathbf{a}^{I}_{I}$ | ◎ | $\mathbf{a}^{I}_{I} = {\mathbf{D}^{I}_{L}}^{T} \cdot \mathbf{a}^{I}_{L}$ | ◎ | $\mathbf{a}^{I}_{L} = (\mathbf{D}^{I}_{L})^{I} \cdot \mathbf{a}^{I}_{I}$ | ◎ | $\mathbf{a}^{I}_{I} = (\mathbf{D}^{I}_{L})^{I^{T}} \cdot \mathbf{a}^{I}_{L}$ | ◎ |
| 2 | $\mathbf{a}^{B}_{L} = \mathbf{D}^{I}_{L} \cdot \mathbf{a}^{B}_{I}$ | × | $\mathbf{a}^{A}_{I} = {\mathbf{D}^{I}_{L}}^{T} \cdot \mathbf{a}^{A}_{L}$ | × | $\mathbf{a}^{B}_{L} = (\mathbf{D}^{I}_{L})^{I} \cdot \mathbf{a}^{B}_{I}$ | × | $\mathbf{a}^{A}_{I} = (\mathbf{D}^{I}_{L})^{I^{T}} \cdot \mathbf{a}^{A}_{L}$ | × |
| 3 | $\mathbf{a}^{I}_{B} = \mathbf{D}^{L}_{B} \cdot \mathrm{a}^{I}_{L}$ | × | $\mathbf{a}^{I}_{L} = {\mathbf{D}^{L}_{B}}^{T} \cdot \mathbf{a}^{I}_{B}$ | × | $\mathbf{a}^{I}_{B} = (\mathbf{D}^{L}_{B})^{I} \cdot \mathbf{a}^{I}_{L}$ | ◎ | $\mathbf{a}^{I}_{L} = (\mathbf{D}^{L}_{B})^{I^{T}} \cdot \mathbf{a}^{I}_{B}$ | ◎ |
| 4 | $\mathbf{a}^{L}_{B} = \mathbf{D}^{L}_{B} \cdot \mathbf{a}^{L}_{L}$ | ◎ | $\mathbf{a}^{L}_{L} = {\mathbf{D}^{L}_{B}}^{T} \cdot \mathbf{a}^{L}_{B}$ | ◎ | $\mathbf{a}^{L}_{B} = (\mathbf{D}^{L}_{B})^{I} \cdot \mathbf{a}^{L}_{L}$ | × | $\mathbf{a}^{L}_{L} = (\mathbf{D}^{L}_{B})^{I^{T}} \cdot \mathbf{a}^{L}_{B}$ | × |
| 5 | $\mathbf{a}^{I}_{A} = \mathbf{D}^{L}_{A} \cdot \mathbf{a}^{I}_{L}$ | × | $\mathbf{a}^{I}_{L} = {\mathbf{D}^{L}_{A}}^{T} \cdot \mathbf{a}^{I}_{A}$ | × | $\mathbf{a}^{I}_{A} = (\mathbf{D}^{L}_{A})^{I} \cdot \mathbf{a}^{I}_{L}$ | ◎ | $\mathbf{a}^{I}_{L} = (\mathbf{D}^{L}_{A})^{I^{T}} \cdot \mathbf{a}^{I}_{A}$ | ◎ |
| 6 | $\mathbf{a}^{L}_{A} = \mathbf{D}^{L}_{A} \cdot \mathbf{a}^{L}_{L}$ | ◎ | $\mathbf{a}^{L}_{L} = {\mathbf{D}^{L}_{A}}^{T} \cdot \mathbf{a}^{L}_{A}$ | ◎ | $\mathbf{a}^{L}_{A} = (\mathbf{D}^{L}_{A})^{I} \cdot \mathbf{a}^{L}_{L}$ | × | $\mathbf{a}^{L}_{L} = (\mathbf{D}^{L}_{A})^{I^{T}} \cdot \mathbf{a}^{L}_{A}$ | × |
| 7 | $\mathbf{a}^{I}_{B} = \mathbf{D}^{I}_{B} \cdot \mathbf{a}^{I}_{I}$ | ◎ | $\mathbf{a}^{I}_{I} = {\mathbf{D}^{I}_{B}}^{T} \cdot \mathbf{a}^{I}_{B}$ | ◎ | $\mathbf{a}^{I}_{B} = (\mathbf{D}^{I}_{B})^{I} \cdot \mathbf{a}^{I}_{I}$ | ◎ | $\mathbf{a}^{I}_{I} = (\mathbf{D}^{I}_{B})^{I^{T}} \cdot \mathbf{a}^{I}_{B}$ | ◎ |
| 8 | $\mathbf{a}^{L}_{B} = \mathbf{D}^{I}_{B} \cdot \mathbf{a}^{L}_{I}$ | × | $\mathbf{a}^{L}_{I} = {\mathbf{D}^{I}_{B}}^{T} \cdot \mathbf{a}^{L}_{B}$ | × | $\mathbf{a}^{L}_{B} = (\mathbf{D}^{I}_{B})^{I} \cdot \mathbf{a}^{L}_{I}$ | × | $\mathbf{a}^{L}_{I} = (\mathbf{D}^{I}_{B})^{I^{T}} \cdot \mathbf{a}^{L}_{B}$ | × |
| 9 | $\mathbf{a}^{I}_{A} = \mathbf{D}^{I}_{A} \cdot \mathbf{a}^{I}_{I}$ | ◎ | $\mathbf{a}^{I}_{I} = {\mathbf{D}^{I}_{A}}^{T} \cdot \mathbf{a}^{I}_{A}$ | ◎ | $\mathbf{a}^{I}_{A} = (\mathbf{D}^{I}_{A})^{I} \cdot \mathbf{a}^{I}_{I}$ | ◎ | $\mathbf{a}^{I}_{I} = (\mathbf{D}^{I}_{A})^{I^{T}} \cdot \mathbf{a}^{I}_{A}$ | ◎ |
| 10 | $\mathbf{a}^{L}_{A} = \mathbf{D}^{I}_{A} \cdot \mathbf{a}^{L}_{I}$ | × | $\mathbf{a}^{L}_{I} = {\mathbf{D}^{I}_{A}}^{T} \cdot \mathbf{a}^{L}_{A}$ | × | $\mathbf{a}^{L}_{A} = (\mathbf{D}^{I}_{A})^{I} \cdot \mathbf{a}^{L}_{I}$ | × | $\mathbf{a}^{L}_{I} = (\mathbf{D}^{I}_{A})^{I^{T}} \cdot \mathbf{a}^{L}_{A}$ | × |
| 11 | $\mathbf{a}^{I}_{A} = \mathbf{D}^{B}_{A} \cdot \mathbf{a}^{I}_{B}$ | × | $\mathbf{a}^{I}_{B} = {\mathbf{D}^{B}_{A}}^{T} \cdot \mathbf{a}^{I}_{A}$ | × | $\mathbf{a}^{I}_{A} = (\mathbf{D}^{B}_{A})^{I} \cdot \mathbf{a}^{I}_{B}$ | ◎ | $\mathbf{a}^{I}_{B} = (\mathbf{D}^{B}_{A})^{I^{T}} \cdot \mathbf{a}^{I}_{A}$ | ◎ |
| 12 | $\mathbf{a}^{L}_{A} = \mathbf{D}^{B}_{A} \cdot \mathbf{a}^{L}_{B}$ | × | $\mathbf{a}^{L}_{B} = {\mathbf{D}^{B}_{A}}^{T} \cdot \mathbf{a}^{L}_{A}$ | × | $\mathbf{a}^{L}_{A} = (\mathbf{D}^{B}_{A})^{I} \cdot \mathbf{a}^{L}_{B}$ | × | $\mathbf{a}^{L}_{B} = (\mathbf{D}^{B}_{A})^{I^{T}} \cdot \mathbf{a}^{L}_{A}$ | × |
| 13 | | | | | $\mathbf{a}^{L}_{A} = (\mathbf{D}^{B}_{A})^{L} \cdot \mathbf{a}^{L}_{B}$ | ◎ | $\mathbf{a}^{L}_{B} = (\mathbf{D}^{B}_{A})^{L^{T}} \cdot \mathbf{a}^{L}_{A}$ | ◎ |

| No. | オイラー角型四元数による回転 | | | | 固定角型四元数による回転 | | | |
|---|---|---|---|---|---|---|---|---|
| 21 | $\mathbf{a}^{I}_{L} = \mathbf{Q}^{I}_{L} \cdot \mathbf{a}^{I}_{I} \cdot {\mathbf{Q}^{I}_{L}}^{*}$ | ◎ | $\mathbf{a}^{I}_{I} = {\mathbf{Q}^{I}_{L}}^{*} \cdot \mathbf{a}^{I}_{L} \cdot \mathbf{Q}^{I}_{L}$ | ◎ | $\mathbf{a}^{I}_{L} = (\mathbf{Q}^{I}_{L})^{I} \cdot \mathbf{a}^{I}_{I} \cdot (\mathbf{Q}^{I}_{L})^{I^{*}}$ | ◎ | $\mathbf{a}^{I}_{I} = (\mathbf{Q}^{I}_{L})^{I^{*}} \cdot \mathbf{a}^{I}_{L} \cdot (\mathbf{Q}^{I}_{L})^{I}$ | ◎ |
| 22 | $\mathbf{a}^{B}_{L} = \mathbf{Q}^{I}_{L} \cdot \mathbf{a}^{B}_{I} \cdot {\mathbf{Q}^{I}_{L}}^{*}$ | × | $\mathbf{a}^{A}_{I} = {\mathbf{Q}^{I}_{L}}^{*} \cdot \mathbf{a}^{A}_{L} \cdot \mathbf{Q}^{I}_{L}$ | × | $\mathbf{a}^{B}_{L} = (\mathbf{Q}^{I}_{L})^{I} \cdot \mathbf{a}^{B}_{I} \cdot (\mathbf{Q}^{I}_{L})^{I^{*}}$ | × | $\mathbf{a}^{A}_{I} = (\mathbf{Q}^{I}_{L})^{I^{*}} \cdot \mathbf{a}^{A}_{L} \cdot (\mathbf{Q}^{I}_{L})^{I}$ | × |
| 23 | $\mathbf{a}^{I}_{B} = \mathbf{Q}^{L}_{B} \cdot \mathbf{a}^{I}_{L} \cdot {\mathbf{Q}^{L}_{B}}^{*}$ | × | $\mathbf{a}^{I}_{L} = {\mathbf{Q}^{L}_{B}}^{*} \cdot \mathbf{a}^{I}_{B} \cdot \mathbf{Q}^{L}_{B}$ | × | $\mathbf{a}^{I}_{B} = (\mathbf{Q}^{L}_{B})^{I} \cdot \mathbf{a}^{I}_{L} \cdot (\mathbf{Q}^{L}_{B})^{I^{*}}$ | ◎ | $\mathbf{a}^{I}_{L} = (\mathbf{Q}^{L}_{B})^{I^{*}} \cdot \mathbf{a}^{I}_{B} \cdot (\mathbf{Q}^{L}_{B})^{I}$ | ◎ |
| 24 | $\mathbf{a}^{L}_{B} = \mathbf{Q}^{L}_{B} \cdot \mathbf{a}^{L}_{L} \cdot {\mathbf{Q}^{L}_{B}}^{*}$ | ◎ | $\mathbf{a}^{L}_{L} = {\mathbf{Q}^{L}_{B}}^{*} \cdot \mathbf{a}^{L}_{B} \cdot \mathbf{Q}^{L}_{B}$ | ◎ | $\mathbf{a}^{L}_{B} = (\mathbf{Q}^{L}_{B})^{I} \cdot \mathbf{a}^{L}_{L} \cdot (\mathbf{Q}^{L}_{B})^{I^{*}}$ | × | $\mathbf{a}^{L}_{L} = (\mathbf{Q}^{L}_{B})^{I^{*}} \cdot \mathbf{a}^{L}_{B} \cdot (\mathbf{Q}^{L}_{B})^{I}$ | × |
| 25 | $\mathbf{a}^{I}_{A} = \mathbf{Q}^{L}_{A} \cdot \mathbf{a}^{I}_{L} \cdot {\mathbf{Q}^{L}_{A}}^{*}$ | × | $\mathbf{a}^{I}_{L} = {\mathbf{Q}^{L}_{A}}^{*} \cdot \mathbf{a}^{I}_{A} \cdot \mathbf{Q}^{L}_{A}$ | × | $\mathbf{a}^{I}_{A} = (\mathbf{Q}^{L}_{A})^{I} \cdot \mathbf{a}^{I}_{L} \cdot (\mathbf{Q}^{L}_{A})^{I^{*}}$ | ◎ | $\mathbf{a}^{I}_{L} = (\mathbf{Q}^{L}_{A})^{I^{*}} \cdot \mathbf{a}^{I}_{A} \cdot (\mathbf{Q}^{L}_{A})^{I}$ | ◎ |
| 26 | $\mathbf{a}^{L}_{A} = \mathbf{Q}^{L}_{A} \cdot \mathbf{a}^{L}_{L} \cdot {\mathbf{Q}^{L}_{A}}^{*}$ | ◎ | $\mathbf{a}^{L}_{L} = {\mathbf{Q}^{L}_{A}}^{*} \cdot \mathbf{a}^{L}_{A} \cdot \mathbf{Q}^{L}_{A}$ | ◎ | $\mathbf{a}^{L}_{A} = (\mathbf{Q}^{L}_{A})^{I} \cdot \mathbf{a}^{L}_{L} \cdot (\mathbf{Q}^{L}_{A})^{I^{*}}$ | × | $\mathbf{a}^{L}_{L} = (\mathbf{Q}^{L}_{A})^{I^{*}} \cdot \mathbf{a}^{L}_{A} \cdot (\mathbf{Q}^{L}_{A})^{I}$ | × |
| 27 | $\mathbf{a}^{I}_{B} = \mathbf{Q}^{I}_{B} \cdot \mathbf{a}^{I}_{I} \cdot {\mathbf{Q}^{I}_{B}}^{*}$ | ◎ | $\mathbf{a}^{I}_{I} = {\mathbf{Q}^{I}_{B}}^{*} \cdot \mathbf{a}^{I}_{B} \cdot \mathbf{Q}^{I}_{B}$ | ◎ | $\mathbf{a}^{I}_{B} = (\mathbf{Q}^{I}_{B})^{I} \cdot \mathbf{a}^{I}_{I} \cdot (\mathbf{Q}^{I}_{B})^{I^{*}}$ | ◎ | $\mathbf{a}^{I}_{I} = (\mathbf{Q}^{I}_{B})^{I^{*}} \cdot \mathbf{a}^{I}_{B} \cdot (\mathbf{Q}^{I}_{B})^{I}$ | ◎ |
| 28 | $\mathbf{a}^{L}_{B} = \mathbf{Q}^{I}_{B} \cdot \mathbf{a}^{L}_{I} \cdot {\mathbf{Q}^{I}_{B}}^{*}$ | × | $\mathbf{a}^{L}_{I} = {\mathbf{Q}^{I}_{B}}^{*} \cdot \mathbf{a}^{L}_{B} \cdot \mathbf{Q}^{I}_{B}$ | × | $\mathbf{a}^{L}_{B} = (\mathbf{Q}^{I}_{B})^{I} \cdot \mathbf{a}^{L}_{I} \cdot (\mathbf{Q}^{I}_{B})^{I^{*}}$ | × | $\mathbf{a}^{L}_{I} = (\mathbf{Q}^{I}_{B})^{I^{*}} \cdot \mathbf{a}^{L}_{B} \cdot (\mathbf{Q}^{I}_{B})^{I}$ | × |
| 29 | $\mathbf{a}^{I}_{A} = \mathbf{Q}^{I}_{A} \cdot \mathbf{a}^{I}_{I} \cdot {\mathbf{Q}^{I}_{A}}^{*}$ | ◎ | $\mathbf{a}^{I}_{I} = {\mathbf{Q}^{I}_{A}}^{*} \cdot \mathbf{a}^{I}_{A} \cdot \mathbf{Q}^{I}_{A}$ | ◎ | $\mathbf{a}^{I}_{A} = (\mathbf{Q}^{I}_{A})^{I} \cdot \mathbf{a}^{I}_{I} \cdot (\mathbf{Q}^{I}_{A})^{I^{*}}$ | ◎ | $\mathbf{a}^{I}_{I} = (\mathbf{Q}^{I}_{A})^{I^{*}} \cdot \mathbf{a}^{I}_{A} \cdot (\mathbf{Q}^{I}_{A})^{I}$ | ◎ |
| 30 | $\mathbf{a}^{L}_{A} = \mathbf{Q}^{I}_{A} \cdot \mathbf{a}^{L}_{I} \cdot {\mathbf{Q}^{I}_{A}}^{*}$ | × | $\mathbf{a}^{L}_{I} = {\mathbf{Q}^{I}_{A}}^{*} \cdot \mathbf{a}^{L}_{A} \cdot \mathbf{Q}^{I}_{A}$ | × | $\mathbf{a}^{L}_{A} = (\mathbf{Q}^{I}_{A})^{I} \cdot \mathbf{a}^{L}_{I} \cdot (\mathbf{Q}^{I}_{A})^{I^{*}}$ | × | $\mathbf{a}^{L}_{I} = (\mathbf{Q}^{I}_{A})^{I^{*}} \cdot \mathbf{a}^{L}_{A} \cdot (\mathbf{Q}^{I}_{A})^{I}$ | × |
| 31 | $\mathbf{a}^{I}_{A} = \mathbf{Q}^{B}_{A} \cdot \mathbf{a}^{I}_{B} \cdot {\mathbf{Q}^{B}_{A}}^{*}$ | × | $\mathbf{a}^{I}_{B} = {\mathbf{Q}^{B}_{A}}^{*} \cdot \mathbf{a}^{I}_{A} \cdot \mathbf{Q}^{B}_{A}$ | × | $\mathbf{a}^{I}_{A} = (\mathbf{Q}^{B}_{A})^{I} \cdot \mathbf{a}^{I}_{B} \cdot (\mathbf{Q}^{B}_{A})^{I^{*}}$ | ◎ | $\mathbf{a}^{I}_{B} = (\mathbf{Q}^{B}_{A})^{I^{*}} \cdot \mathbf{a}^{I}_{A} \cdot (\mathbf{Q}^{B}_{A})^{I}$ | ◎ |
| 32 | $\mathbf{a}^{L}_{A} = \mathbf{Q}^{B}_{A} \cdot \mathbf{a}^{L}_{B} \cdot {\mathbf{Q}^{B}_{A}}^{*}$ | × | $\mathbf{a}^{L}_{B} = {\mathbf{Q}^{B}_{A}}^{*} \cdot \mathbf{a}^{L}_{A} \cdot \mathbf{Q}^{B}_{A}$ | × | $\mathbf{a}^{L}_{A} = (\mathbf{Q}^{B}_{A})^{I} \cdot \mathbf{a}^{L}_{B} \cdot (\mathbf{Q}^{B}_{A})^{I^{*}}$ | × | $\mathbf{a}^{L}_{B} = (\mathbf{Q}^{B}_{A})^{I^{*}} \cdot \mathbf{a}^{L}_{A} \cdot (\mathbf{Q}^{B}_{A})^{I}$ | × |
| 33 | | | | | $\mathbf{a}^{L}_{A} = (\mathbf{Q}^{B}_{A})^{L} \cdot \mathbf{a}^{L}_{B} \cdot (\mathbf{Q}^{B}_{A})^{L^{*}}$ | ◎ | $\mathbf{a}^{L}_{B} = (\mathbf{Q}^{B}_{A})^{L^{*}} \cdot \mathbf{a}^{L}_{A} \cdot (\mathbf{Q}^{B}_{A})^{L}$ | ◎ |

　ベクトルの回転は固定角型回転マトリクスおよび固定角型四元数が利用できるが、基準の座標系が一致していなければならない。

任意のベクトルの回転は次のように、「慣性航法のアルゴリズム」に示した式(1.4-6)で表される。

$$\mathbf{b}^{R} = \left(\mathbf{D}^{A}_{G}\right)^{R} \cdot \mathbf{a}^{R}$$ ：ベクトルの回転

$$\mathbf{a}^{R}_{G} = \left(\mathbf{D}^{A}_{G}\right)^{R} \cdot \mathbf{a}^{R}_{A}$$ ：座標系Rで表した座標系Aの軸ベクトルを座標系Gへ回転

$\mathbf{a}^{R}, \mathbf{b}^{R}$ ：座標系Rで表したベクトル

$\left(\mathbf{D}_{G}^{A}\right)^{R} = \mathbf{D}_{G}^{R} \cdot \mathbf{D}_{A}^{R\,T}$ ：座標系で表した座標系Aから座標系Gへの固定角型回転マトリクス

$\mathbf{D}_{G}^{R} = [\theta_1]_i \cdot [\theta_2]_j \cdots [\theta_n]_k$ ：座標系RからG（座標系でなくても良い）への回転を表すオイラー角

$\mathbf{D}_{A}^{R} = [\varphi_1]_e \cdot [\varphi_2]_f \cdots [\varphi_m]_g$ ：座標系RからA（座標系でなくても良い）への回転を表すオイラー角

方向余弦マトリクスでも同様に、方向余弦マトリクスの右上添え字側から右下添え字側に向かって回転される。

$\mathbf{b}^{R} = \mathbf{D}_{G}^{R} \cdot \mathbf{a}^{R}$ ：座標系Rで表したベクトルのRからGへの回転

(1) 方向余弦マトリクスによるベクトルの回転

$$\mathbf{a}_{L}^{I} = \mathbf{D}_{L}^{I} \cdot \mathbf{a}_{I}^{I} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{I}^{I\,T} \cdot \mathbf{a}_{I}^{I} = \left(\mathbf{D}_{L}^{I}\right)^{I} \cdot \mathbf{a}_{I}^{I} = \mathbf{a}_{L}^{I}$$

この方向余弦マトリクスは固定角型回転マトリクスと等しく、座標系Iで表された任意のベクトルに対し、座標系Iにおいて座標系Iから座標系Lへの回転と同じ回転を与える。ここでは回転の量が分かるように座標軸ベクトルを対象としたので右下添え字に座標系の記号を示しているが、基準の座標系が一致する任意のベクトルが回転できる。

$$\mathbf{a}_{B}^{L} = \mathbf{D}_{B}^{L} \cdot \mathbf{a}_{L}^{L} = \mathbf{D}_{B}^{L} \cdot \mathbf{D}_{L}^{L\,T} \cdot \mathbf{a}_{L}^{L} = \left(\mathbf{D}_{B}^{L}\right)^{L} \cdot \mathbf{a}_{L}^{L} = \mathbf{a}_{B}^{L}$$
$$\mathbf{a}_{A}^{L} = \mathbf{D}_{A}^{L} \cdot \mathbf{a}_{L}^{L} = \mathbf{D}_{A}^{L} \cdot \mathbf{D}_{L}^{L\,T} \cdot \mathbf{a}_{L}^{L} = \left(\mathbf{D}_{A}^{L}\right)^{L} \cdot \mathbf{a}_{L}^{L} = \mathbf{a}_{A}^{L}$$
$$\mathbf{a}_{B}^{I} = \mathbf{D}_{B}^{I} \cdot \mathbf{a}_{I}^{I} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{I}^{I\,T} \cdot \mathbf{a}_{I}^{I} = \left(\mathbf{D}_{B}^{I}\right)^{I} \cdot \mathbf{a}_{I}^{I} = \mathbf{a}_{B}^{I}$$
$$\mathbf{a}_{A}^{I} = \mathbf{D}_{A}^{I} \cdot \mathbf{a}_{I}^{I} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{I}^{I\,T} \cdot \mathbf{a}_{I}^{I} = \left(\mathbf{D}_{A}^{I}\right)^{I} \cdot \mathbf{a}_{I}^{I} = \mathbf{a}_{A}^{I}$$
$$\mathbf{a}_{I}^{I} = \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{L}^{I} = \mathbf{D}_{I}^{I} \cdot \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{L}^{I} = \left(\mathbf{D}_{I}^{L}\right)^{I} \cdot \mathbf{a}_{L}^{I} = \mathbf{a}_{I}^{I}$$
$$\mathbf{a}_{L}^{L} = \mathbf{D}_{B}^{L\,T} \cdot \mathbf{a}_{B}^{L} = \mathbf{D}_{L}^{L} \cdot \mathbf{D}_{B}^{L\,T} \cdot \mathbf{a}_{B}^{L} = \left(\mathbf{D}_{L}^{B}\right)^{L} \cdot \mathbf{a}_{B}^{L} = \mathbf{a}_{L}^{L}$$
$$\mathbf{a}_{L}^{L} = \mathbf{D}_{A}^{L\,T} \cdot \mathbf{a}_{A}^{L} = \mathbf{D}_{L}^{L} \cdot \mathbf{D}_{A}^{L\,T} \cdot \mathbf{a}_{A}^{L} = \left(\mathbf{D}_{L}^{A}\right)^{L} \cdot \mathbf{a}_{A}^{L} = \mathbf{a}_{L}^{L}$$
$$\mathbf{a}_{I}^{I} = \mathbf{D}_{B}^{I\,T} \cdot \mathbf{a}_{B}^{I} = \mathbf{D}_{I}^{I} \cdot \mathbf{D}_{B}^{I\,T} \cdot \mathbf{a}_{B}^{I} = \left(\mathbf{D}_{I}^{B}\right)^{I} \cdot \mathbf{a}_{B}^{I} = \mathbf{a}_{I}^{I}$$
$$\mathbf{a}_{I}^{I} = \mathbf{D}_{A}^{I\,T} \cdot \mathbf{a}_{A}^{I} = \mathbf{D}_{I}^{I} \cdot \mathbf{D}_{A}^{I\,T} \cdot \mathbf{a}_{A}^{I} = \left(\mathbf{D}_{I}^{A}\right)^{I} \cdot \mathbf{a}_{A}^{I} = \mathbf{a}_{I}^{I}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_{L}^{B} = \mathbf{D}_{L}^{I} \cdot \mathbf{a}_{I}^{B} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{I}^{I\,T} \cdot \mathbf{a}_{I}^{B} = \left(\mathbf{D}_{L}^{I}\right)^{I} \cdot \mathbf{a}_{I}^{B}$$
$$\mathbf{a}_{B}^{I} = \mathbf{D}_{B}^{L} \cdot \mathbf{a}_{L}^{I} = \mathbf{D}_{B}^{L} \cdot \mathbf{D}_{L}^{L\,T} \cdot \mathbf{a}_{L}^{I} = \left(\mathbf{D}_{B}^{L}\right)^{L} \cdot \mathbf{a}_{L}^{I}$$
$$\mathbf{a}_{A}^{I} = \mathbf{D}_{A}^{L} \cdot \mathbf{a}_{L}^{I} = \mathbf{D}_{A}^{L} \cdot \mathbf{D}_{L}^{L\,T} \cdot \mathbf{a}_{L}^{I} = \left(\mathbf{D}_{A}^{L}\right)^{L} \cdot \mathbf{a}_{L}^{I}$$
$$\mathbf{a}_{B}^{L} = \mathbf{D}_{B}^{I} \cdot \mathbf{a}_{I}^{L} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{I}^{I\,T} \cdot \mathbf{a}_{I}^{L} = \left(\mathbf{D}_{B}^{I}\right)^{I} \cdot \mathbf{a}_{I}^{L}$$
$$\mathbf{a}_{A}^{L} = \mathbf{D}_{A}^{I} \cdot \mathbf{a}_{I}^{L} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{I}^{I\,T} \cdot \mathbf{a}_{I}^{L} = \left(\mathbf{D}_{A}^{I}\right)^{I} \cdot \mathbf{a}_{I}^{L}$$
$$\mathbf{a}_{A}^{I} = \mathbf{D}_{A}^{B} \cdot \mathbf{a}_{B}^{I} = \mathbf{D}_{A}^{B} \cdot \mathbf{D}_{B}^{B\,T} \cdot \mathbf{a}_{B}^{I} = \left(\mathbf{D}_{A}^{B}\right)^{B} \cdot \mathbf{a}_{B}^{I}$$
$$\mathbf{a}_{A}^{L} = \mathbf{D}_{A}^{B} \cdot \mathbf{a}_{B}^{L} = \mathbf{D}_{A}^{B} \cdot \mathbf{D}_{B}^{B\,T} \cdot \mathbf{a}_{B}^{L} = \left(\mathbf{D}_{A}^{B}\right)^{B} \cdot \mathbf{a}_{B}^{L}$$
$$\mathbf{a}_{I}^{A} = \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{I}^{I} \cdot \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{L}^{A} = \left(\mathbf{D}_{I}^{L}\right)^{I} \cdot \mathbf{a}_{L}^{A}$$
$$\mathbf{a}_{L}^{I} = \mathbf{D}_{B}^{L\,T} \cdot \mathbf{a}_{B}^{I} = \mathbf{D}_{L}^{L} \cdot \mathbf{D}_{B}^{L\,T} \cdot \mathbf{a}_{B}^{I} = \left(\mathbf{D}_{L}^{B}\right)^{L} \cdot \mathbf{a}_{B}^{I}$$
$$\mathbf{a}_{L}^{I} = \mathbf{D}_{A}^{L\,T} \cdot \mathbf{a}_{A}^{I} = \mathbf{D}_{L}^{L} \cdot \mathbf{D}_{A}^{L\,T} \cdot \mathbf{a}_{A}^{I} = \left(\mathbf{D}_{L}^{A}\right)^{L} \cdot \mathbf{a}_{A}^{I}$$
$$\mathbf{a}_{I}^{L} = \mathbf{D}_{B}^{I\,T} \cdot \mathbf{a}_{B}^{L} = \mathbf{D}_{I}^{I} \cdot \mathbf{D}_{B}^{I\,T} \cdot \mathbf{a}_{B}^{L} = \left(\mathbf{D}_{I}^{B}\right)^{I} \cdot \mathbf{a}_{B}^{L}$$
$$\mathbf{a}_{I}^{L} = \mathbf{D}_{A}^{I\,T} \cdot \mathbf{a}_{A}^{L} = \mathbf{D}_{I}^{I} \cdot \mathbf{D}_{A}^{I\,T} \cdot \mathbf{a}_{A}^{L} = \left(\mathbf{D}_{I}^{A}\right)^{I} \cdot \mathbf{a}_{A}^{L}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

(2) 固定角型回転マトリクスによるベクトルの回転

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

(3) オイラー角型四元数によるベクトルの回転

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$

$$\mathbf{a}_A^I = \mathbf{Q}_A^I \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_A^{I\,*} = \mathbf{D}_A^I \cdot \mathbf{a}_I^I = \mathbf{D}_A^I \cdot \mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^I = \left(\mathbf{D}_A^I\right)^I \cdot \mathbf{a}_I^I = \mathbf{a}_A^I$$
$$\mathbf{a}_I^I = \mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_L^I = \mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^I = \mathbf{D}_I^I \cdot \mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^I = \left(\mathbf{D}_I^L\right)^I \cdot \mathbf{a}_L^I = \mathbf{a}_I^I$$
$$\mathbf{a}_L^L = \mathbf{Q}_B^{L\,*} \cdot \mathbf{a}_B^L \cdot \mathbf{Q}_B^L = \mathbf{D}_B^{L\,T} \cdot \mathbf{a}_B^L = \mathbf{D}_L^L \cdot \mathbf{D}_B^{L\,T} \cdot \mathbf{a}_B^L = \left(\mathbf{D}_L^B\right)^L \cdot \mathbf{a}_B^L = \mathbf{a}_L^L$$
$$\mathbf{a}_L^L = \mathbf{Q}_A^{L\,*} \cdot \mathbf{a}_A^L \cdot \mathbf{Q}_A^L = \mathbf{D}_A^{L\,T} \cdot \mathbf{a}_A^L = \mathbf{D}_L^L \cdot \mathbf{D}_A^{L\,T} \cdot \mathbf{a}_A^L = \left(\mathbf{D}_L^A\right)^L \cdot \mathbf{a}_A^L = \mathbf{a}_L^L$$
$$\mathbf{a}_I^I = \mathbf{Q}_B^{I\,*} \cdot \mathbf{a}_B^I \cdot \mathbf{Q}_B^I = \mathbf{D}_B^{I\,T} \cdot \mathbf{a}_B^I = \mathbf{D}_I^I \cdot \mathbf{D}_B^{I\,T} \cdot \mathbf{a}_B^I = \left(\mathbf{D}_I^B\right)^I \cdot \mathbf{a}_B^I = \mathbf{a}_I^I$$
$$\mathbf{a}_I^I = \mathbf{Q}_A^{I\,*} \cdot \mathbf{a}_A^I \cdot \mathbf{Q}_A^I = \mathbf{D}_A^{I\,T} \cdot \mathbf{a}_A^I = \mathbf{D}_I^I \cdot \mathbf{D}_A^{I\,T} \cdot \mathbf{a}_A^I = \left(\mathbf{D}_I^A\right)^I \cdot \mathbf{a}_A^I = \mathbf{a}_I^I$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_L^B = \mathbf{Q}_L^I \cdot \mathbf{a}_I^B \cdot \mathbf{Q}_L^{I\,*} = \mathbf{D}_L^I \cdot \mathbf{a}_I^B = \mathbf{D}_L^I \cdot \mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^B = \left(\mathbf{D}_L^I\right)^I \cdot \mathbf{a}_I^B$$
$$\mathbf{a}_B^I = \mathbf{Q}_B^L \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_B^{L\,*} = \mathbf{D}_B^L \cdot \mathbf{a}_L^I = \mathbf{D}_B^L \cdot \mathbf{D}_L^{L\,T} \cdot \mathbf{a}_L^I = \left(\mathbf{D}_B^L\right)^L \cdot \mathbf{a}_L^I$$
$$\mathbf{a}_A^I = \mathbf{Q}_A^L \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_A^{L\,*} = \mathbf{D}_A^L \cdot \mathbf{a}_L^I = \mathbf{D}_A^L \cdot \mathbf{D}_L^{L\,T} \cdot \mathbf{a}_L^I = \left(\mathbf{D}_A^L\right)^L \cdot \mathbf{a}_L^I$$
$$\mathbf{a}_B^L = \mathbf{Q}_B^I \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_B^{I\,*} = \mathbf{D}_B^I \cdot \mathbf{a}_I^L = \mathbf{D}_B^I \cdot \mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^L = \left(\mathbf{D}_B^I\right)^I \cdot \mathbf{a}_I^L$$
$$\mathbf{a}_A^L = \mathbf{Q}_A^I \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_A^{I\,*} = \mathbf{D}_A^I \cdot \mathbf{a}_I^L = \mathbf{D}_A^I \cdot \mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^L = \left(\mathbf{D}_A^I\right)^I \cdot \mathbf{a}_I^L$$
$$\mathbf{a}_A^I = \mathbf{Q}_A^B \cdot \mathbf{a}_B^I \cdot \mathbf{Q}_A^{B\,*} = \mathbf{D}_A^B \cdot \mathbf{a}_B^I = \mathbf{D}_A^B \cdot \mathbf{D}_B^{B\,T} \cdot \mathbf{a}_B^I = \left(\mathbf{D}_A^B\right)^B \cdot \mathbf{a}_B^I$$
$$\mathbf{a}_A^L = \mathbf{Q}_A^B \cdot \mathbf{a}_B^L \cdot \mathbf{Q}_A^{B\,*} = \mathbf{D}_A^B \cdot \mathbf{a}_B^L = \mathbf{D}_A^B \cdot \mathbf{D}_B^{B\,T} \cdot \mathbf{a}_B^L = \left(\mathbf{D}_A^B\right)^B \cdot \mathbf{a}_B^L$$
$$\mathbf{a}_I^A = \mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^A \cdot \mathbf{Q}_L^I = \mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^A = \mathbf{D}_I^I \cdot \mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^A = \left(\mathbf{D}_I^L\right)^I \cdot \mathbf{a}_L^A$$
$$\mathbf{a}_L^I = \mathbf{Q}_B^{L\,*} \cdot \mathbf{a}_B^I \cdot \mathbf{Q}_B^L = \mathbf{D}_B^{L\,T} \cdot \mathbf{a}_B^I = \mathbf{D}_L^L \cdot \mathbf{D}_B^{L\,T} \cdot \mathbf{a}_B^I = \left(\mathbf{D}_L^B\right)^L \cdot \mathbf{a}_B^I$$
$$\mathbf{a}_L^I = \mathbf{Q}_A^{L\,*} \cdot \mathbf{a}_A^I \cdot \mathbf{Q}_A^L = \mathbf{D}_A^{L\,T} \cdot \mathbf{a}_A^I = \mathbf{D}_L^L \cdot \mathbf{D}_A^{L\,T} \cdot \mathbf{a}_A^I = \left(\mathbf{D}_L^A\right)^L \cdot \mathbf{a}_A^I$$
$$\mathbf{a}_I^L = \mathbf{Q}_B^{I\,*} \cdot \mathbf{a}_B^L \cdot \mathbf{Q}_B^I = \mathbf{D}_B^{I\,T} \cdot \mathbf{a}_B^L = \mathbf{D}_I^I \cdot \mathbf{D}_B^{I\,T} \cdot \mathbf{a}_B^L = \left(\mathbf{D}_I^B\right)^I \cdot \mathbf{a}_B^L$$
$$\mathbf{a}_I^L = \mathbf{Q}_A^{I\,*} \cdot \mathbf{a}_A^L \cdot \mathbf{Q}_A^I = \mathbf{D}_A^{I\,T} \cdot \mathbf{a}_A^L = \mathbf{D}_I^I \cdot \mathbf{D}_A^{I\,T} \cdot \mathbf{a}_A^L = \left(\mathbf{D}_I^A\right)^I \cdot \mathbf{a}_A^L$$
$$\mathbf{a}_B^I = \mathbf{Q}_A^{B\,*} \cdot \mathbf{a}_A^I \cdot \mathbf{Q}_A^B = \mathbf{D}_A^{B\,T} \cdot \mathbf{a}_A^I = \mathbf{D}_B^B \cdot \mathbf{D}_A^{B\,T} \cdot \mathbf{a}_A^I = \left(\mathbf{D}_B^A\right)^B \cdot \mathbf{a}_A^I$$
$$\mathbf{a}_B^L = \mathbf{Q}_A^{B\,*} \cdot \mathbf{a}_A^L \cdot \mathbf{Q}_A^B = \mathbf{D}_A^{B\,T} \cdot \mathbf{a}_A^L = \mathbf{D}_B^B \cdot \mathbf{D}_A^{B\,T} \cdot \mathbf{a}_A^L = \left(\mathbf{D}_B^A\right)^B \cdot \mathbf{a}_A^L$$

(4) 固定角型四元数によるベクトルの回転

$$\begin{aligned}
\mathbf{a}_L^I &= \left(\mathbf{Q}_L^I\right)^I \cdot \mathbf{a}_I^I \cdot \left(\mathbf{Q}_L^I\right)^{I\,*} = \mathbf{Q}_L^I \cdot \mathbf{Q}_I^{I\,*} \cdot \mathbf{a}_I^I \cdot \left(\mathbf{Q}_L^I \cdot \mathbf{Q}_I^{I\,*}\right)^* = \mathbf{Q}_L^I \cdot \mathbf{Q}_I^{I\,*} \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_I^I \cdot \mathbf{Q}_L^{I\,*} \\
&= \mathbf{Q}_L^I \cdot \left(\mathbf{Q}_I^{I\,*} \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_I^I\right) \cdot \mathbf{Q}_L^{I\,*} = \mathbf{D}_L^I \cdot \left(\mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^I\right) = \mathbf{D}_L^I \cdot \mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^I = \left(\mathbf{D}_L^I\right)^I \cdot \mathbf{a}_I^I = \mathbf{a}_L^I \\
\mathbf{a}_B^I &= \left(\mathbf{Q}_B^L\right)^I \cdot \mathbf{a}_L^I \cdot \left(\mathbf{Q}_B^L\right)^{I\,*} = \mathbf{Q}_B^I \cdot \mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^I \cdot \left(\mathbf{Q}_B^I \cdot \mathbf{Q}_L^{I\,*}\right)^* = \mathbf{Q}_B^I \cdot \mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_L^I \cdot \mathbf{Q}_B^{I\,*} \\
&= \mathbf{Q}_B^I \cdot \left(\mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_L^I\right) \cdot \mathbf{Q}_B^{I\,*} = \mathbf{D}_B^I \cdot \left(\mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^I\right) = \mathbf{D}_B^I \cdot \mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^I = \left(\mathbf{D}_B^L\right)^I \cdot \mathbf{a}_L^I = \mathbf{a}_B^I \\
\mathbf{a}_A^I &= \left(\mathbf{Q}_A^L\right)^I \cdot \mathbf{a}_L^I \cdot \left(\mathbf{Q}_A^L\right)^{I\,*} = \mathbf{Q}_A^I \cdot \mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^I \cdot \left(\mathbf{Q}_A^I \cdot \mathbf{Q}_L^{I\,*}\right)^* = \mathbf{Q}_A^I \cdot \mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_L^I \cdot \mathbf{Q}_A^{I\,*} \\
&= \mathbf{Q}_A^I \cdot \left(\mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_L^I\right) \cdot \mathbf{Q}_A^{I\,*} = \mathbf{D}_A^I \cdot \left(\mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^I\right) = \mathbf{D}_A^I \cdot \mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^I = \left(\mathbf{D}_A^L\right)^I \cdot \mathbf{a}_L^I = \mathbf{a}_A^I \\
\mathbf{a}_B^I &= \left(\mathbf{Q}_B^I\right)^I \cdot \mathbf{a}_I^I \cdot \left(\mathbf{Q}_B^I\right)^{I\,*} = \mathbf{Q}_B^I \cdot \mathbf{Q}_I^{I\,*} \cdot \mathbf{a}_I^I \cdot \left(\mathbf{Q}_B^I \cdot \mathbf{Q}_I^{I\,*}\right)^* = \mathbf{Q}_B^I \cdot \mathbf{Q}_I^{I\,*} \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_I^I \cdot \mathbf{Q}_B^{I\,*} \\
&= \mathbf{Q}_B^I \cdot \left(\mathbf{Q}_I^{I\,*} \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_I^I\right) \cdot \mathbf{Q}_B^{I\,*} = \mathbf{D}_B^I \cdot \left(\mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^I\right) = \mathbf{D}_B^I \cdot \mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^I = \left(\mathbf{D}_B^I\right)^I \cdot \mathbf{a}_I^I = \mathbf{a}_B^I \\
\mathbf{a}_A^I &= \left(\mathbf{Q}_A^I\right)^I \cdot \mathbf{a}_I^I \cdot \left(\mathbf{Q}_A^I\right)^{I\,*} = \mathbf{Q}_A^I \cdot \mathbf{Q}_I^{I\,*} \cdot \mathbf{a}_I^I \cdot \left(\mathbf{Q}_A^I \cdot \mathbf{Q}_I^{I\,*}\right)^* = \mathbf{Q}_A^I \cdot \mathbf{Q}_I^{I\,*} \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_I^I \cdot \mathbf{Q}_A^{I\,*} \\
&= \mathbf{Q}_A^I \cdot \left(\mathbf{Q}_I^{I\,*} \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_I^I\right) \cdot \mathbf{Q}_A^{I\,*} = \mathbf{D}_A^I \cdot \left(\mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^I\right) = \mathbf{D}_A^I \cdot \mathbf{D}_I^{I\,T} \cdot \mathbf{a}_I^I = \left(\mathbf{D}_A^I\right)^I \cdot \mathbf{a}_I^I = \mathbf{a}_A^I \\
\mathbf{a}_A^I &= \left(\mathbf{Q}_A^B\right)^I \cdot \mathbf{a}_B^I \cdot \left(\mathbf{Q}_A^B\right)^{I\,*} = \mathbf{Q}_A^I \cdot \mathbf{Q}_B^{I\,*} \cdot \mathbf{a}_B^I \cdot \left(\mathbf{Q}_A^I \cdot \mathbf{Q}_B^{I\,*}\right)^* = \mathbf{Q}_A^I \cdot \mathbf{Q}_B^{I\,*} \cdot \mathbf{a}_B^I \cdot \mathbf{Q}_B^I \cdot \mathbf{Q}_A^{I\,*} \\
&= \mathbf{Q}_A^I \cdot \left(\mathbf{Q}_B^{I\,*} \cdot \mathbf{a}_B^I \cdot \mathbf{Q}_B^I\right) \cdot \mathbf{Q}_A^{I\,*} = \mathbf{D}_A^I \cdot \left(\mathbf{D}_B^{I\,T} \cdot \mathbf{a}_B^I\right) = \mathbf{D}_A^I \cdot \mathbf{D}_B^{I\,T} \cdot \mathbf{a}_B^I = \left(\mathbf{D}_A^B\right)^I \cdot \mathbf{a}_B^I = \mathbf{a}_A^I \\
\mathbf{a}_A^L &= \left(\mathbf{Q}_A^B\right)^L \cdot \mathbf{a}_B^L \cdot \left(\mathbf{Q}_A^B\right)^{L\,*} = \mathbf{Q}_A^L \cdot \mathbf{Q}_B^{L\,*} \cdot \mathbf{a}_B^L \cdot \left(\mathbf{Q}_A^L \cdot \mathbf{Q}_B^{L\,*}\right)^* = \mathbf{Q}_A^L \cdot \mathbf{Q}_B^{L\,*} \cdot \mathbf{a}_B^L \cdot \mathbf{Q}_B^L \cdot \mathbf{Q}_A^{L\,*} \\
&= \mathbf{Q}_A^L \cdot \left(\mathbf{Q}_B^{L\,*} \cdot \mathbf{a}_B^L \cdot \mathbf{Q}_B^L\right) \cdot \mathbf{Q}_A^{L\,*} = \mathbf{D}_A^L \cdot \left(\mathbf{D}_B^{L\,T} \cdot \mathbf{a}_B^L\right) = \mathbf{D}_A^L \cdot \mathbf{D}_B^{L\,T} \cdot \mathbf{a}_B^L = \left(\mathbf{D}_A^B\right)^L \cdot \mathbf{a}_B^L = \mathbf{a}_A^L
\end{aligned}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*}\right)^* \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*}$$

$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$

$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}{}^{*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}{}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}}{}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}{}^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}$$

$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}{}^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}{}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}}{}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}{}^{*}\right)^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}{}^{*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}{}^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}{}^{*}$$

$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}{}^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}{}^{*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}{}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}}{}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

表5-3　ベクトルの座標変換

| No. | 方向余弦マトリクスによる座標変換 | | | | 固定角型回転マトリクスによる座標変換 | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | $\mathbf{a}^{I}_{B}=\mathbf{D}^{I}_{L}\cdot\mathbf{a}^{L}_{B}$ | ◎ | $\mathbf{a}^{L}_{B}=\mathbf{D}^{I\,T}_{L}\cdot\mathbf{a}^{I}_{B}$ | ◎ | $\mathbf{a}^{I}_{B}=(\mathbf{D}^{I}_{L})^{I}\cdot\mathbf{a}^{L}_{B}$ | ◎ | $\mathbf{a}^{L}_{B}=(\mathbf{D}^{I}_{L})^{I\,T}\cdot\mathbf{a}^{I}_{B}$ | ◎ |
| 2 | $\mathbf{a}^{I}_{A}=\mathbf{D}^{I}_{L}\cdot\mathbf{a}^{L}_{A}$ | ◎ | $\mathbf{a}^{L}_{A}=\mathbf{D}^{I\,T}_{L}\cdot\mathbf{a}^{I}_{A}$ | ◎ | $\mathbf{a}^{I}_{A}=(\mathbf{D}^{I}_{L})^{I}\cdot\mathbf{a}^{L}_{A}$ | ◎ | $\mathbf{a}^{L}_{A}=(\mathbf{D}^{I}_{L})^{I\,T}\cdot\mathbf{a}^{I}_{A}$ | ◎ |
| 3 | $\mathbf{a}^{L}_{I}=\mathbf{D}^{L}_{B}\cdot\mathbf{a}^{B}_{I}$ | ◎ | $\mathbf{a}^{B}_{I}=\mathbf{D}^{L\,T}_{B}\cdot\mathbf{a}^{L}_{I}$ | ◎ | $\mathbf{a}^{L}_{I}=(\mathbf{D}^{L}_{B})^{I}\cdot\mathbf{a}^{B}_{I}$ | × | $\mathbf{a}^{B}_{I}=(\mathbf{D}^{L}_{B})^{I\,T}\cdot\mathbf{a}^{L}_{I}$ | × |
| 4 | $\mathbf{a}^{L}_{L}=\mathbf{D}^{L}_{B}\cdot\mathbf{a}^{B}_{L}$ | ◎ | $\mathbf{a}^{B}_{L}=\mathbf{D}^{L\,T}_{B}\cdot\mathbf{a}^{L}_{L}$ | ◎ | $\mathbf{a}^{L}_{L}=(\mathbf{D}^{L}_{B})^{I}\cdot\mathbf{a}^{B}_{L}$ | × | $\mathbf{a}^{B}_{L}=(\mathbf{D}^{L}_{B})^{I\,T}\cdot\mathbf{a}^{L}_{L}$ | × |
| 5 | $\mathbf{a}^{L}_{I}=\mathbf{D}^{L}_{A}\cdot\mathbf{a}^{A}_{I}$ | ◎ | $\mathbf{a}^{A}_{I}=\mathbf{D}^{L\,T}_{A}\cdot\mathbf{a}^{L}_{I}$ | ◎ | $\mathbf{a}^{L}_{I}=(\mathbf{D}^{L}_{A})^{I}\cdot\mathbf{a}^{A}_{I}$ | × | $\mathbf{a}^{A}_{I}=(\mathbf{D}^{L}_{A})^{I\,T}\cdot\mathbf{a}^{L}_{I}$ | × |
| 6 | $\mathbf{a}^{L}_{L}=\mathbf{D}^{L}_{A}\cdot\mathbf{a}^{A}_{L}$ | ◎ | $\mathbf{a}^{A}_{L}=\mathbf{D}^{L\,T}_{A}\cdot\mathbf{a}^{L}_{L}$ | ◎ | $\mathbf{a}^{L}_{L}=(\mathbf{D}^{L}_{A})^{I}\cdot\mathbf{a}^{A}_{L}$ | × | $\mathbf{a}^{A}_{L}=(\mathbf{D}^{L}_{A})^{I\,T}\cdot\mathbf{a}^{L}_{L}$ | × |
| 7 | $\mathbf{a}^{I}_{I}=\mathbf{D}^{I}_{B}\cdot\mathbf{a}^{B}_{I}$ | ◎ | $\mathbf{a}^{B}_{I}=\mathbf{D}^{I\,T}_{B}\cdot\mathbf{a}^{I}_{I}$ | ◎ | $\mathbf{a}^{I}_{I}=(\mathbf{D}^{I}_{B})^{I}\cdot\mathbf{a}^{B}_{I}$ | ◎ | $\mathbf{a}^{B}_{I}=(\mathbf{D}^{I}_{B})^{I\,T}\cdot\mathbf{a}^{I}_{I}$ | ◎ |
| 8 | $\mathbf{a}^{I}_{L}=\mathbf{D}^{I}_{B}\cdot\mathbf{a}^{B}_{L}$ | ◎ | $\mathbf{a}^{B}_{L}=\mathbf{D}^{I\,T}_{B}\cdot\mathbf{a}^{I}_{L}$ | ◎ | $\mathbf{a}^{I}_{L}=(\mathbf{D}^{I}_{B})^{I}\cdot\mathbf{a}^{B}_{L}$ | ◎ | $\mathbf{a}^{B}_{L}=(\mathbf{D}^{I}_{B})^{I\,T}\cdot\mathbf{a}^{I}_{L}$ | ◎ |
| 9 | $\mathbf{a}^{I}_{I}=\mathbf{D}^{I}_{A}\cdot\mathbf{a}^{A}_{I}$ | ◎ | $\mathbf{a}^{A}_{I}=\mathbf{D}^{I\,T}_{A}\cdot\mathbf{a}^{I}_{I}$ | ◎ | $\mathbf{a}^{I}_{I}=(\mathbf{D}^{I}_{A})^{I}\cdot\mathbf{a}^{A}_{I}$ | ◎ | $\mathbf{a}^{A}_{I}=(\mathbf{D}^{I}_{A})^{I\,T}\cdot\mathbf{a}^{I}_{I}$ | ◎ |
| 10 | $\mathbf{a}^{I}_{L}=\mathbf{D}^{I}_{A}\cdot\mathbf{a}^{A}_{L}$ | ◎ | $\mathbf{a}^{A}_{L}=\mathbf{D}^{I\,T}_{A}\cdot\mathbf{a}^{I}_{L}$ | ◎ | $\mathbf{a}^{I}_{L}=(\mathbf{D}^{I}_{A})^{I}\cdot\mathbf{a}^{A}_{L}$ | ◎ | $\mathbf{a}^{A}_{L}=(\mathbf{D}^{I}_{A})^{I\,T}\cdot\mathbf{a}^{I}_{L}$ | ◎ |
| 11 | $\mathbf{a}^{B}_{I}=\mathbf{D}^{B}_{A}\cdot\mathbf{a}^{A}_{I}$ | ◎ | $\mathbf{a}^{A}_{I}=\mathbf{D}^{B\,T}_{A}\cdot\mathbf{a}^{B}_{I}$ | ◎ | $\mathbf{a}^{B}_{I}=(\mathbf{D}^{B}_{A})^{I}\cdot\mathbf{a}^{A}_{I}$ | × | $\mathbf{a}^{A}_{I}=(\mathbf{D}^{B}_{A})^{I\,T}\cdot\mathbf{a}^{B}_{I}$ | × |
| 12 | $\mathbf{a}^{B}_{L}=\mathbf{D}^{B}_{A}\cdot\mathbf{a}^{A}_{L}$ | ◎ | $\mathbf{a}^{A}_{L}=\mathbf{D}^{B\,T}_{A}\cdot\mathbf{a}^{B}_{L}$ | ◎ | $\mathbf{a}^{B}_{L}=(\mathbf{D}^{B}_{A})^{I}\cdot\mathbf{a}^{A}_{L}$ | × | $\mathbf{a}^{A}_{L}=(\mathbf{D}^{B}_{A})^{I\,T}\cdot\mathbf{a}^{B}_{L}$ | × |
| 13 | | | | | $\mathbf{a}^{B}_{L}=(\mathbf{D}^{B}_{A})^{L}\cdot\mathbf{a}^{A}_{L}$ | × | $\mathbf{a}^{A}_{L}=(\mathbf{D}^{B}_{A})^{L\,T}\cdot\mathbf{a}^{B}_{L}$ | × |

| No. | オイラー角型四元数による座標変換 | | | | 固定角型四元数による座標変換 | | | |
|---|---|---|---|---|---|---|---|---|
| 21 | $\mathbf{a}^{I}_{B}=\mathbf{Q}^{I}_{L}\cdot\mathbf{a}^{L}_{B}\cdot\mathbf{Q}^{I\,*}_{L}$ | ◎ | $\mathbf{a}^{L}_{B}=\mathbf{Q}^{I\,*}_{L}\cdot\mathbf{a}^{I}_{B}\cdot\mathbf{Q}^{I}_{L}$ | ◎ | $\mathbf{a}^{I}_{B}=(\mathbf{Q}^{I}_{L})^{I}\cdot\mathbf{a}^{L}_{B}\cdot(\mathbf{Q}^{I}_{L})^{I^*}$ | ◎ | $\mathbf{a}^{L}_{B}=(\mathbf{Q}^{I}_{L})^{I^*}\cdot\mathbf{a}^{I}_{B}\cdot(\mathbf{Q}^{I}_{L})^{I}$ | ◎ |
| 22 | $\mathbf{a}^{I}_{A}=\mathbf{Q}^{I}_{L}\cdot\mathbf{a}^{L}_{A}\cdot\mathbf{Q}^{I\,*}_{L}$ | ◎ | $\mathbf{a}^{L}_{A}=\mathbf{Q}^{I\,*}_{L}\cdot\mathbf{a}^{I}_{A}\cdot\mathbf{Q}^{I}_{L}$ | ◎ | $\mathbf{a}^{I}_{A}=(\mathbf{Q}^{I}_{L})^{I}\cdot\mathbf{a}^{L}_{A}\cdot(\mathbf{Q}^{I}_{L})^{I^*}$ | ◎ | $\mathbf{a}^{L}_{A}=(\mathbf{Q}^{I}_{L})^{I^*}\cdot\mathbf{a}^{I}_{A}\cdot(\mathbf{Q}^{I}_{L})^{I}$ | ◎ |
| 23 | $\mathbf{a}^{L}_{I}=\mathbf{Q}^{L}_{B}\cdot\mathbf{a}^{B}_{I}\cdot\mathbf{Q}^{L\,*}_{B}$ | ◎ | $\mathbf{a}^{B}_{I}=\mathbf{Q}^{L\,*}_{B}\cdot\mathbf{a}^{L}_{I}\cdot\mathbf{Q}^{L}_{B}$ | ◎ | $\mathbf{a}^{L}_{I}=(\mathbf{Q}^{L}_{B})^{I}\cdot\mathbf{a}^{B}_{I}\cdot(\mathbf{Q}^{L}_{B})^{I^*}$ | × | $\mathbf{a}^{B}_{I}=(\mathbf{Q}^{L}_{B})^{I^*}\cdot\mathbf{a}^{L}_{I}\cdot(\mathbf{Q}^{L}_{B})^{I}$ | × |
| 24 | $\mathbf{a}^{L}_{L}=\mathbf{Q}^{L}_{B}\cdot\mathbf{a}^{B}_{L}\cdot\mathbf{Q}^{L\,*}_{B}$ | ◎ | $\mathbf{a}^{B}_{L}=\mathbf{Q}^{L\,*}_{B}\cdot\mathbf{a}^{L}_{L}\cdot\mathbf{Q}^{L}_{B}$ | ◎ | $\mathbf{a}^{L}_{L}=(\mathbf{Q}^{L}_{B})^{I}\cdot\mathbf{a}^{B}_{L}\cdot(\mathbf{Q}^{L}_{B})^{I^*}$ | × | $\mathbf{a}^{B}_{L}=(\mathbf{Q}^{L}_{B})^{I^*}\cdot\mathbf{a}^{L}_{L}\cdot(\mathbf{Q}^{L}_{B})^{I}$ | × |
| 25 | $\mathbf{a}^{L}_{I}=\mathbf{Q}^{L}_{A}\cdot\mathbf{a}^{A}_{I}\cdot\mathbf{Q}^{L\,*}_{A}$ | ◎ | $\mathbf{a}^{A}_{I}=\mathbf{Q}^{L\,*}_{A}\cdot\mathbf{a}^{L}_{I}\cdot\mathbf{Q}^{L}_{A}$ | ◎ | $\mathbf{a}^{L}_{I}=(\mathbf{Q}^{L}_{A})^{I}\cdot\mathbf{a}^{A}_{I}\cdot(\mathbf{Q}^{L}_{A})^{I^*}$ | × | $\mathbf{a}^{A}_{I}=(\mathbf{Q}^{L}_{A})^{I^*}\cdot\mathbf{a}^{L}_{I}\cdot(\mathbf{Q}^{L}_{A})^{I}$ | × |
| 26 | $\mathbf{a}^{L}_{L}=\mathbf{Q}^{L}_{A}\cdot\mathbf{a}^{A}_{L}\cdot\mathbf{Q}^{L\,*}_{A}$ | ◎ | $\mathbf{a}^{A}_{L}=\mathbf{Q}^{L\,*}_{A}\cdot\mathbf{a}^{L}_{L}\cdot\mathbf{Q}^{L}_{A}$ | ◎ | $\mathbf{a}^{L}_{L}=(\mathbf{Q}^{L}_{A})^{I}\cdot\mathbf{a}^{A}_{L}\cdot(\mathbf{Q}^{L}_{A})^{I^*}$ | × | $\mathbf{a}^{A}_{L}=(\mathbf{Q}^{L}_{A})^{I^*}\cdot\mathbf{a}^{L}_{L}\cdot(\mathbf{Q}^{L}_{A})^{I}$ | × |
| 27 | $\mathbf{a}^{I}_{I}=\mathbf{Q}^{I}_{B}\cdot\mathbf{a}^{B}_{I}\cdot\mathbf{Q}^{I\,*}_{B}$ | ◎ | $\mathbf{a}^{B}_{I}=\mathbf{Q}^{I\,*}_{B}\cdot\mathbf{a}^{I}_{I}\cdot\mathbf{Q}^{I}_{B}$ | ◎ | $\mathbf{a}^{I}_{I}=(\mathbf{Q}^{I}_{B})^{I}\cdot\mathbf{a}^{B}_{I}\cdot(\mathbf{Q}^{I}_{B})^{I^*}$ | ◎ | $\mathbf{a}^{B}_{I}=(\mathbf{Q}^{I}_{B})^{I^*}\cdot\mathbf{a}^{I}_{I}\cdot(\mathbf{Q}^{I}_{B})^{I}$ | ◎ |
| 28 | $\mathbf{a}^{I}_{L}=\mathbf{Q}^{I}_{B}\cdot\mathbf{a}^{B}_{L}\cdot\mathbf{Q}^{I\,*}_{B}$ | ◎ | $\mathbf{a}^{B}_{L}=\mathbf{Q}^{I\,*}_{B}\cdot\mathbf{a}^{I}_{L}\cdot\mathbf{Q}^{I}_{B}$ | ◎ | $\mathbf{a}^{I}_{L}=(\mathbf{Q}^{I}_{B})^{I}\cdot\mathbf{a}^{B}_{L}\cdot(\mathbf{Q}^{I}_{B})^{I^*}$ | ◎ | $\mathbf{a}^{B}_{L}=(\mathbf{Q}^{I}_{B})^{I^*}\cdot\mathbf{a}^{I}_{L}\cdot(\mathbf{Q}^{I}_{B})^{I}$ | ◎ |
| 29 | $\mathbf{a}^{I}_{I}=\mathbf{Q}^{I}_{A}\cdot\mathbf{a}^{A}_{I}\cdot\mathbf{Q}^{I\,*}_{A}$ | ◎ | $\mathbf{a}^{A}_{I}=\mathbf{Q}^{I\,*}_{A}\cdot\mathbf{a}^{I}_{I}\cdot\mathbf{Q}^{I}_{A}$ | ◎ | $\mathbf{a}^{I}_{I}=(\mathbf{Q}^{I}_{A})^{I}\cdot\mathbf{a}^{A}_{I}\cdot(\mathbf{Q}^{I}_{A})^{I^*}$ | ◎ | $\mathbf{a}^{A}_{I}=(\mathbf{Q}^{I}_{A})^{I^*}\cdot\mathbf{a}^{I}_{I}\cdot(\mathbf{Q}^{I}_{A})^{I}$ | ◎ |
| 30 | $\mathbf{a}^{I}_{L}=\mathbf{Q}^{I}_{A}\cdot\mathbf{a}^{A}_{L}\cdot\mathbf{Q}^{I\,*}_{A}$ | ◎ | $\mathbf{a}^{A}_{L}=\mathbf{Q}^{I\,*}_{A}\cdot\mathbf{a}^{I}_{L}\cdot\mathbf{Q}^{I}_{A}$ | ◎ | $\mathbf{a}^{I}_{L}=(\mathbf{Q}^{I}_{A})^{I}\cdot\mathbf{a}^{A}_{L}\cdot(\mathbf{Q}^{I}_{A})^{I^*}$ | ◎ | $\mathbf{a}^{A}_{L}=(\mathbf{Q}^{I}_{A})^{I^*}\cdot\mathbf{a}^{I}_{L}\cdot(\mathbf{Q}^{I}_{A})^{I}$ | ◎ |
| 31 | $\mathbf{a}^{B}_{I}=\mathbf{Q}^{B}_{A}\cdot\mathbf{a}^{A}_{I}\cdot\mathbf{Q}^{B\,*}_{A}$ | ◎ | $\mathbf{a}^{A}_{I}=\mathbf{Q}^{B\,*}_{A}\cdot\mathbf{a}^{B}_{I}\cdot\mathbf{Q}^{B}_{A}$ | ◎ | $\mathbf{a}^{B}_{I}=(\mathbf{Q}^{B}_{A})^{I}\cdot\mathbf{a}^{A}_{I}\cdot(\mathbf{Q}^{B}_{A})^{I^*}$ | × | $\mathbf{a}^{A}_{I}=(\mathbf{Q}^{B}_{A})^{I^*}\cdot\mathbf{a}^{B}_{I}\cdot(\mathbf{Q}^{B}_{A})^{I}$ | × |
| 32 | $\mathbf{a}^{B}_{L}=\mathbf{Q}^{B}_{A}\cdot\mathbf{a}^{A}_{L}\cdot\mathbf{Q}^{B\,*}_{A}$ | ◎ | $\mathbf{a}^{A}_{L}=\mathbf{Q}^{B\,*}_{A}\cdot\mathbf{a}^{B}_{L}\cdot\mathbf{Q}^{B}_{A}$ | ◎ | $\mathbf{a}^{B}_{L}=(\mathbf{Q}^{B}_{A})^{I}\cdot\mathbf{a}^{A}_{L}\cdot(\mathbf{Q}^{B}_{A})^{I^*}$ | × | $\mathbf{a}^{A}_{L}=(\mathbf{Q}^{B}_{A})^{I^*}\cdot\mathbf{a}^{B}_{L}\cdot(\mathbf{Q}^{B}_{A})^{I}$ | × |
| 33 | | | | | $\mathbf{a}^{B}_{L}=(\mathbf{Q}^{B}_{A})^{L}\cdot\mathbf{a}^{A}_{L}\cdot(\mathbf{Q}^{B}_{A})^{L^*}$ | × | $\mathbf{a}^{A}_{L}=(\mathbf{Q}^{B}_{A})^{L^*}\cdot\mathbf{a}^{B}_{L}\cdot(\mathbf{Q}^{B}_{A})^{L}$ | × |

ベクトルの座標変換は方向余弦マトリクスまたはオイラー角型四元数を用いて、基準座標系の如何に依らず変換できる。

方向余弦マトリクスによる座標系の変換は、右下添え字側の座標系から右上添え字側の座標系へ切り替わる。座標系Gから見たベクトルを座標系Aから見たベクトルに座標変換するのは次のようになる。

$$\mathbf{a}^{A}=\mathbf{D}^{A}_{G}\cdot\mathbf{a}^{G}$$ :ベクトルの座標変換

(1) 方向余弦マトリクスによる座標変換

ベクトルの座標変換の式は次のように証明される。

$$\mathbf{a}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot \mathbf{D}^{\mathrm{L}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{B}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot \mathbf{D}^{\mathrm{L}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{A}}$$
$$\mathbf{a}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{B}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{B}} \cdot \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{L}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{B}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{B}} \cdot \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{L}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{A}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{A}} \cdot \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{L}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{A}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{A}} \cdot \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{L}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{A}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{A}} \cdot \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{B}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{A}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{A}} \cdot \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{B}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{L}}_{\mathrm{B}} = {\mathbf{D}^{\mathrm{I}}_{\mathrm{L}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{B}} = \mathbf{a}^{\mathrm{L}}_{\mathrm{B}}$$
$$\mathbf{a}^{\mathrm{L}}_{\mathrm{A}} = {\mathbf{D}^{\mathrm{I}}_{\mathrm{L}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{A}} = \mathbf{a}^{\mathrm{L}}_{\mathrm{A}}$$
$$\mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = {\mathbf{D}^{\mathrm{L}}_{\mathrm{B}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} \cdot \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{B}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = {\mathbf{D}^{\mathrm{L}}_{\mathrm{B}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} \cdot \mathbf{D}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{B}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = {\mathbf{D}^{\mathrm{L}}_{\mathrm{A}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} \cdot \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{A}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{A}}_{\mathrm{L}} = {\mathbf{D}^{\mathrm{L}}_{\mathrm{A}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} \cdot \mathbf{D}^{\mathrm{L}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{A}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = {\mathbf{D}^{\mathrm{I}}_{\mathrm{B}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{B}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = {\mathbf{D}^{\mathrm{I}}_{\mathrm{B}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{B}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = {\mathbf{D}^{\mathrm{I}}_{\mathrm{A}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{A}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{A}}_{\mathrm{L}} = {\mathbf{D}^{\mathrm{I}}_{\mathrm{A}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{A}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = {\mathbf{D}^{\mathrm{B}}_{\mathrm{A}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{B}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{B}} \cdot \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{A}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{A}}_{\mathrm{L}} = {\mathbf{D}^{\mathrm{B}}_{\mathrm{A}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{B}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{B}} \cdot \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{A}}_{\mathrm{L}}$$

(2) 固定角型回転マトリクスによる座標変換

$$\mathbf{a}^{\mathrm{I}}_{\mathrm{B}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot \mathbf{D}^{\mathrm{L}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{B}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{A}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot \mathbf{a}^{\mathrm{L}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot \mathbf{D}^{\mathrm{L}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{A}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot \mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot \mathbf{a}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot \mathbf{D}^{\mathrm{A}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{I}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{L}}_{\mathrm{B}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{L}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{B}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{L}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{B}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{B}} = \mathbf{a}^{\mathrm{L}}_{\mathrm{B}}$$
$$\mathbf{a}^{\mathrm{L}}_{\mathrm{A}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{L}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{A}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{L}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{L}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{A}} = \mathbf{D}^{\mathrm{L}}_{\mathrm{A}} = \mathbf{a}^{\mathrm{L}}_{\mathrm{A}}$$
$$\mathbf{a}^{\mathrm{B}}_{\mathrm{I}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{B}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{B}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{B}}_{\mathrm{I}}$$
$$\mathbf{a}^{\mathrm{B}}_{\mathrm{L}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{B}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{B}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{B}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{L}} = \mathbf{D}^{\mathrm{B}}_{\mathrm{L}} = \mathbf{a}^{\mathrm{B}}_{\mathrm{L}}$$
$$\mathbf{a}^{\mathrm{A}}_{\mathrm{I}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{A}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \left(\mathbf{D}^{\mathrm{I}}_{\mathrm{A}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{I}}}^{\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} \cdot {\mathbf{D}^{\mathrm{I}}_{\mathrm{A}}}^{\mathrm{T}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} \cdot \mathbf{a}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} \cdot \mathbf{D}^{\mathrm{I}}_{\mathrm{I}} = \mathbf{D}^{\mathrm{A}}_{\mathrm{I}} = \mathbf{a}^{\mathrm{A}}_{\mathrm{I}}$$

$$\mathbf{a}_{L}^{A} = \left(\mathbf{D}_{A}^{I}\right)^{I\,T} \cdot \mathbf{a}_{L}^{I} = \left(\mathbf{D}_{A}^{I} \cdot \mathbf{D}_{I}^{I\,T}\right)^{T} \cdot \mathbf{a}_{L}^{I} = \mathbf{D}_{I}^{I} \cdot \mathbf{D}_{A}^{I\,T} \cdot \mathbf{a}_{L}^{I} = \mathbf{D}_{I}^{A} \cdot \mathbf{a}_{L}^{I} = \mathbf{D}_{I}^{A} \cdot \mathbf{D}_{L}^{I} = \mathbf{D}_{L}^{A} = \mathbf{a}_{L}^{A}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_{I}^{L} = \left(\mathbf{D}_{B}^{L}\right)^{I} \cdot \mathbf{a}_{I}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{I}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{I}^{L} \cdot \mathbf{D}_{I}^{B}$$
$$\mathbf{a}_{L}^{L} = \left(\mathbf{D}_{B}^{L}\right)^{I} \cdot \mathbf{a}_{L}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{L}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{I}^{L} \cdot \mathbf{D}_{L}^{B}$$
$$\mathbf{a}_{I}^{L} = \left(\mathbf{D}_{A}^{L}\right)^{I} \cdot \mathbf{a}_{I}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{I}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{I}^{L} \cdot \mathbf{D}_{I}^{A}$$
$$\mathbf{a}_{L}^{L} = \left(\mathbf{D}_{A}^{L}\right)^{I} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{I}^{L} \cdot \mathbf{D}_{L}^{A}$$
$$\mathbf{a}_{I}^{B} = \left(\mathbf{D}_{A}^{B}\right)^{I} \cdot \mathbf{a}_{I}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{B}^{I\,T} \cdot \mathbf{a}_{I}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{I}^{B} \cdot \mathbf{D}_{I}^{A}$$
$$\mathbf{a}_{L}^{B} = \left(\mathbf{D}_{A}^{B}\right)^{I} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{B}^{I\,T} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{I}^{B} \cdot \mathbf{D}_{L}^{A}$$
$$\mathbf{a}_{L}^{B} = \left(\mathbf{D}_{A}^{B}\right)^{L} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{A}^{L} \cdot \mathbf{D}_{B}^{L\,T} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{A}^{L} \cdot \mathbf{D}_{L}^{B} \cdot \mathbf{D}_{L}^{A}$$
$$\mathbf{a}_{I}^{B} = \left(\mathbf{D}_{B}^{L}\right)^{I\,T} \cdot \mathbf{a}_{I}^{L} = \left(\mathbf{D}_{B}^{I} \cdot \mathbf{D}_{L}^{I\,T}\right)^{T} \cdot \mathbf{a}_{I}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{B}^{I\,T} \cdot \mathbf{a}_{I}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{I}^{B} \cdot \mathbf{D}_{I}^{L}$$
$$\mathbf{a}_{L}^{B} = \left(\mathbf{D}_{B}^{L}\right)^{I\,T} \cdot \mathbf{a}_{L}^{L} = \left(\mathbf{D}_{B}^{I} \cdot \mathbf{D}_{L}^{I\,T}\right)^{T} \cdot \mathbf{a}_{L}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{B}^{I\,T} \cdot \mathbf{a}_{L}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{I}^{B} \cdot \mathbf{D}_{L}^{L}$$
$$\mathbf{a}_{I}^{A} = \left(\mathbf{D}_{A}^{L}\right)^{I\,T} \cdot \mathbf{a}_{I}^{L} = \left(\mathbf{D}_{A}^{I} \cdot \mathbf{D}_{L}^{I\,T}\right)^{T} \cdot \mathbf{a}_{I}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{A}^{I\,T} \cdot \mathbf{a}_{I}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{I}^{A} \cdot \mathbf{D}_{I}^{L}$$
$$\mathbf{a}_{L}^{A} = \left(\mathbf{D}_{A}^{L}\right)^{I\,T} \cdot \mathbf{a}_{L}^{L} = \left(\mathbf{D}_{A}^{I} \cdot \mathbf{D}_{L}^{I\,T}\right)^{T} \cdot \mathbf{a}_{L}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{A}^{I\,T} \cdot \mathbf{a}_{L}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{I}^{A} \cdot \mathbf{D}_{L}^{L}$$
$$\mathbf{a}_{I}^{A} = \left(\mathbf{D}_{A}^{B}\right)^{I\,T} \cdot \mathbf{a}_{I}^{B} = \left(\mathbf{D}_{A}^{I} \cdot \mathbf{D}_{B}^{I\,T}\right)^{T} \cdot \mathbf{a}_{I}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{A}^{I\,T} \cdot \mathbf{a}_{I}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{I}^{A} \cdot \mathbf{D}_{I}^{B}$$
$$\mathbf{a}_{L}^{A} = \left(\mathbf{D}_{A}^{B}\right)^{I\,T} \cdot \mathbf{a}_{L}^{B} = \left(\mathbf{D}_{A}^{I} \cdot \mathbf{D}_{B}^{I\,T}\right)^{T} \cdot \mathbf{a}_{L}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{A}^{I\,T} \cdot \mathbf{a}_{L}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{I}^{A} \cdot \mathbf{D}_{L}^{B}$$
$$\mathbf{a}_{L}^{A} = \left(\mathbf{D}_{A}^{B}\right)^{L\,T} \cdot \mathbf{a}_{L}^{B} = \left(\mathbf{D}_{A}^{L} \cdot \mathbf{D}_{B}^{L\,T}\right)^{T} \cdot \mathbf{a}_{L}^{B} = \mathbf{D}_{B}^{L} \cdot \mathbf{D}_{A}^{L\,T} \cdot \mathbf{a}_{L}^{B} = \mathbf{D}_{B}^{L} \cdot \mathbf{D}_{L}^{A} \cdot \mathbf{D}_{L}^{B}$$

(3) オイラー角型四元数による座標変換

$$\mathbf{a}_{B}^{I} = \mathbf{Q}_{L}^{I} \cdot \mathbf{a}_{B}^{L} \cdot \mathbf{Q}_{L}^{I\,*} = \mathbf{D}_{L}^{I} \cdot \mathbf{a}_{B}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{B}^{L} = \mathbf{D}_{B}^{I} = \mathbf{a}_{B}^{I}$$
$$\mathbf{a}_{A}^{I} = \mathbf{Q}_{L}^{I} \cdot \mathbf{a}_{A}^{L} \cdot \mathbf{Q}_{L}^{I\,*} = \mathbf{D}_{L}^{I} \cdot \mathbf{a}_{A}^{L} = \mathbf{D}_{L}^{I} \cdot \mathbf{D}_{A}^{L} = \mathbf{D}_{A}^{I} = \mathbf{a}_{A}^{I}$$
$$\mathbf{a}_{I}^{L} = \mathbf{Q}_{B}^{L} \cdot \mathbf{a}_{I}^{B} \cdot \mathbf{Q}_{B}^{L\,*} = \mathbf{D}_{B}^{L} \cdot \mathbf{a}_{I}^{B} = \mathbf{D}_{B}^{L} \cdot \mathbf{D}_{I}^{B} = \mathbf{D}_{I}^{L} = \mathbf{a}_{I}^{L}$$
$$\mathbf{a}_{L}^{L} = \mathbf{Q}_{B}^{L} \cdot \mathbf{a}_{L}^{B} \cdot \mathbf{Q}_{B}^{L\,*} = \mathbf{D}_{B}^{L} \cdot \mathbf{a}_{L}^{B} = \mathbf{D}_{B}^{L} \cdot \mathbf{D}_{L}^{B} = \mathbf{D}_{L}^{L} = \mathbf{a}_{L}^{L}$$
$$\mathbf{a}_{I}^{L} = \mathbf{Q}_{A}^{L} \cdot \mathbf{a}_{I}^{A} \cdot \mathbf{Q}_{A}^{L\,*} = \mathbf{D}_{A}^{L} \cdot \mathbf{a}_{I}^{A} = \mathbf{D}_{A}^{L} \cdot \mathbf{D}_{I}^{A} = \mathbf{D}_{I}^{L} = \mathbf{a}_{I}^{L}$$
$$\mathbf{a}_{L}^{L} = \mathbf{Q}_{A}^{L} \cdot \mathbf{a}_{L}^{A} \cdot \mathbf{Q}_{A}^{L\,*} = \mathbf{D}_{A}^{L} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{A}^{L} \cdot \mathbf{D}_{L}^{A} = \mathbf{D}_{L}^{L} = \mathbf{a}_{L}^{L}$$
$$\mathbf{a}_{I}^{I} = \mathbf{Q}_{B}^{I} \cdot \mathbf{a}_{I}^{B} \cdot \mathbf{Q}_{B}^{I\,*} = \mathbf{D}_{B}^{I} \cdot \mathbf{a}_{I}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{I}^{B} = \mathbf{D}_{I}^{I} = \mathbf{a}_{I}^{I}$$
$$\mathbf{a}_{L}^{I} = \mathbf{Q}_{B}^{I} \cdot \mathbf{a}_{L}^{B} \cdot \mathbf{Q}_{B}^{I\,*} = \mathbf{D}_{B}^{I} \cdot \mathbf{a}_{L}^{B} = \mathbf{D}_{B}^{I} \cdot \mathbf{D}_{L}^{B} = \mathbf{D}_{L}^{I} = \mathbf{a}_{L}^{I}$$
$$\mathbf{a}_{I}^{I} = \mathbf{Q}_{A}^{I} \cdot \mathbf{a}_{I}^{A} \cdot \mathbf{Q}_{A}^{I\,*} = \mathbf{D}_{A}^{I} \cdot \mathbf{a}_{I}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{I}^{A} = \mathbf{D}_{I}^{I} = \mathbf{a}_{I}^{I}$$
$$\mathbf{a}_{L}^{I} = \mathbf{Q}_{A}^{I} \cdot \mathbf{a}_{L}^{A} \cdot \mathbf{Q}_{A}^{I\,*} = \mathbf{D}_{A}^{I} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{A}^{I} \cdot \mathbf{D}_{L}^{A} = \mathbf{D}_{L}^{I} = \mathbf{a}_{L}^{I}$$
$$\mathbf{a}_{I}^{B} = \mathbf{Q}_{A}^{B} \cdot \mathbf{a}_{I}^{A} \cdot \mathbf{Q}_{A}^{B\,*} = \mathbf{D}_{A}^{B} \cdot \mathbf{a}_{I}^{A} = \mathbf{D}_{A}^{B} \cdot \mathbf{D}_{I}^{A} = \mathbf{D}_{I}^{B} = \mathbf{a}_{I}^{B}$$
$$\mathbf{a}_{L}^{B} = \mathbf{Q}_{A}^{B} \cdot \mathbf{a}_{L}^{A} \cdot \mathbf{Q}_{A}^{B\,*} = \mathbf{D}_{A}^{B} \cdot \mathbf{a}_{L}^{A} = \mathbf{D}_{A}^{B} \cdot \mathbf{D}_{L}^{A} = \mathbf{D}_{L}^{B} = \mathbf{a}_{L}^{B}$$
$$\mathbf{a}_{B}^{L} = \mathbf{Q}_{L}^{I\,*} \cdot \mathbf{a}_{B}^{I} \cdot \mathbf{Q}_{L}^{I} = \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{B}^{I} = \mathbf{D}_{I}^{L} \cdot \mathbf{D}_{B}^{I} = \mathbf{D}_{B}^{L} = \mathbf{a}_{B}^{L}$$
$$\mathbf{a}_{A}^{L} = \mathbf{Q}_{L}^{I\,*} \cdot \mathbf{a}_{A}^{I} \cdot \mathbf{Q}_{L}^{I} = \mathbf{D}_{L}^{I\,T} \cdot \mathbf{a}_{A}^{I} = \mathbf{D}_{I}^{L} \cdot \mathbf{D}_{A}^{I} = \mathbf{D}_{A}^{L} = \mathbf{a}_{A}^{L}$$
$$\mathbf{a}_{I}^{B} = \mathbf{Q}_{B}^{L\,*} \cdot \mathbf{a}_{I}^{L} \cdot \mathbf{Q}_{B}^{L} = \mathbf{D}_{B}^{L\,T} \cdot \mathbf{a}_{I}^{L} = \mathbf{D}_{L}^{B} \cdot \mathbf{D}_{I}^{L} = \mathbf{D}_{I}^{B} = \mathbf{a}_{I}^{B}$$
$$\mathbf{a}_{L}^{B} = \mathbf{Q}_{B}^{L\,*} \cdot \mathbf{a}_{L}^{L} \cdot \mathbf{Q}_{B}^{L} = \mathbf{D}_{B}^{L\,T} \cdot \mathbf{a}_{L}^{L} = \mathbf{D}_{L}^{B} \cdot \mathbf{D}_{L}^{L} = \mathbf{D}_{L}^{B} = \mathbf{a}_{L}^{B}$$
$$\mathbf{a}_{I}^{A} = \mathbf{Q}_{A}^{L\,*} \cdot \mathbf{a}_{I}^{L} \cdot \mathbf{Q}_{A}^{L} = \mathbf{D}_{A}^{L\,T} \cdot \mathbf{a}_{I}^{L} = \mathbf{D}_{L}^{A} \cdot \mathbf{D}_{I}^{L} = \mathbf{D}_{I}^{A} = \mathbf{a}_{I}^{A}$$
$$\mathbf{a}_{L}^{A} = \mathbf{Q}_{A}^{L\,*} \cdot \mathbf{a}_{L}^{L} \cdot \mathbf{Q}_{A}^{L} = \mathbf{D}_{A}^{L\,T} \cdot \mathbf{a}_{L}^{L} = \mathbf{D}_{L}^{A} \cdot \mathbf{D}_{L}^{L} = \mathbf{D}_{L}^{A} = \mathbf{a}_{L}^{A}$$
$$\mathbf{a}_{I}^{B} = \mathbf{Q}_{B}^{I\,*} \cdot \mathbf{a}_{I}^{I} \cdot \mathbf{Q}_{B}^{I} = \mathbf{D}_{B}^{I\,T} \cdot \mathbf{a}_{I}^{I} = \mathbf{D}_{I}^{B} \cdot \mathbf{D}_{I}^{I} = \mathbf{D}_{I}^{B} = \mathbf{a}_{I}^{B}$$

$$\mathbf{a}_\mathrm{L}^\mathrm{B} = {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{B}^\mathrm{I} = {\mathbf{D}_\mathrm{B}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{B} \cdot \mathbf{D}_\mathrm{L}^\mathrm{I} = \mathbf{D}_\mathrm{L}^\mathrm{B} = \mathbf{a}_\mathrm{L}^\mathrm{B}$$
$$\mathbf{a}_\mathrm{I}^\mathrm{A} = {\mathbf{Q}_\mathrm{A}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{I} = {\mathbf{D}_\mathrm{A}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{A} \cdot \mathbf{D}_\mathrm{I}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{A} = \mathbf{a}_\mathrm{I}^\mathrm{A}$$
$$\mathbf{a}_\mathrm{L}^\mathrm{A} = {\mathbf{Q}_\mathrm{A}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{I} = {\mathbf{D}_\mathrm{A}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{A} \cdot \mathbf{D}_\mathrm{L}^\mathrm{I} = \mathbf{D}_\mathrm{L}^\mathrm{A} = \mathbf{a}_\mathrm{L}^\mathrm{A}$$
$$\mathbf{a}_\mathrm{I}^\mathrm{A} = {\mathbf{Q}_\mathrm{A}^\mathrm{B}}^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{B} = {\mathbf{D}_\mathrm{A}^\mathrm{B}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} = \mathbf{D}_\mathrm{B}^\mathrm{A} \cdot \mathbf{D}_\mathrm{I}^\mathrm{B} = \mathbf{D}_\mathrm{I}^\mathrm{A} = \mathbf{a}_\mathrm{I}^\mathrm{A}$$
$$\mathbf{a}_\mathrm{L}^\mathrm{A} = {\mathbf{Q}_\mathrm{A}^\mathrm{B}}^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{B} = {\mathbf{D}_\mathrm{A}^\mathrm{B}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} = \mathbf{D}_\mathrm{B}^\mathrm{A} \cdot \mathbf{D}_\mathrm{L}^\mathrm{B} = \mathbf{D}_\mathrm{L}^\mathrm{A} = \mathbf{a}_\mathrm{L}^\mathrm{A}$$

(4) 固定角型四元数による座標変換

$$\mathbf{a}_\mathrm{B}^\mathrm{I} = \left(\mathbf{Q}_\mathrm{L}^\mathrm{I}\right)^\mathrm{I} \cdot \mathbf{a}_\mathrm{B}^\mathrm{L} \cdot \left(\mathbf{Q}_\mathrm{L}^\mathrm{I}\right)^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{B}^\mathrm{L} \cdot \left(\mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{B}^\mathrm{L} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^*$$
$$= \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{B}^\mathrm{L} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I}\right) \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* = \mathbf{D}_\mathrm{L}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{B}^\mathrm{L}\right) = \mathbf{D}_\mathrm{L}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{B}^\mathrm{L} = \mathbf{D}_\mathrm{L}^\mathrm{I} \cdot \mathbf{D}_\mathrm{B}^\mathrm{L} = \mathbf{D}_\mathrm{B}^\mathrm{I} = \mathbf{a}_\mathrm{B}^\mathrm{I}$$
$$\mathbf{a}_\mathrm{A}^\mathrm{I} = \left(\mathbf{Q}_\mathrm{L}^\mathrm{I}\right)^\mathrm{I} \cdot \mathbf{a}_\mathrm{A}^\mathrm{L} \cdot \left(\mathbf{Q}_\mathrm{L}^\mathrm{I}\right)^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{A}^\mathrm{L} \cdot \left(\mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{A}^\mathrm{L} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^*$$
$$= \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{A}^\mathrm{L} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I}\right) \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* = \mathbf{D}_\mathrm{L}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{A}^\mathrm{L}\right) = \mathbf{D}_\mathrm{L}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{A}^\mathrm{L} = \mathbf{D}_\mathrm{L}^\mathrm{I} \cdot \mathbf{D}_\mathrm{B}^\mathrm{L} = \mathbf{D}_\mathrm{B}^\mathrm{I} = \mathbf{a}_\mathrm{B}^\mathrm{I}$$
$$\mathbf{a}_\mathrm{I}^\mathrm{I} = \left(\mathbf{Q}_\mathrm{B}^\mathrm{I}\right)^\mathrm{I} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{I}\right)^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^*$$
$$= \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I}\right) \cdot {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^* = \mathbf{D}_\mathrm{B}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B}\right) = \mathbf{D}_\mathrm{B}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} = \mathbf{D}_\mathrm{B}^\mathrm{I} \cdot \mathbf{D}_\mathrm{I}^\mathrm{B} = \mathbf{D}_\mathrm{I}^\mathrm{I} = \mathbf{a}_\mathrm{I}^\mathrm{I}$$
$$\mathbf{a}_\mathrm{L}^\mathrm{I} = \left(\mathbf{Q}_\mathrm{B}^\mathrm{I}\right)^\mathrm{I} \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{I}\right)^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^*$$
$$= \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I}\right) \cdot {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^* = \mathbf{D}_\mathrm{B}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{B}\right) = \mathbf{D}_\mathrm{B}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} = \mathbf{D}_\mathrm{B}^\mathrm{I} \cdot \mathbf{D}_\mathrm{L}^\mathrm{B} = \mathbf{D}_\mathrm{L}^\mathrm{I} = \mathbf{a}_\mathrm{L}^\mathrm{I}$$
$$\mathbf{a}_\mathrm{I}^\mathrm{I} = \left(\mathbf{Q}_\mathrm{A}^\mathrm{I}\right)^\mathrm{I} \cdot \mathbf{a}_\mathrm{I}^\mathrm{A} \cdot \left(\mathbf{Q}_\mathrm{A}^\mathrm{I}\right)^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{I}^\mathrm{A} \cdot \left(\mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{I}^\mathrm{A} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{A}^\mathrm{I}}^*$$
$$= \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{I}^\mathrm{A} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I}\right) \cdot {\mathbf{Q}_\mathrm{A}^\mathrm{I}}^* = \mathbf{D}_\mathrm{A}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{A}\right) = \mathbf{D}_\mathrm{A}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{A} = \mathbf{D}_\mathrm{A}^\mathrm{I} \cdot \mathbf{D}_\mathrm{I}^\mathrm{A} = \mathbf{D}_\mathrm{I}^\mathrm{I} = \mathbf{a}_\mathrm{I}^\mathrm{I}$$
$$\mathbf{a}_\mathrm{L}^\mathrm{I} = \left(\mathbf{Q}_\mathrm{A}^\mathrm{I}\right)^\mathrm{I} \cdot \mathbf{a}_\mathrm{L}^\mathrm{A} \cdot \left(\mathbf{Q}_\mathrm{A}^\mathrm{I}\right)^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{L}^\mathrm{A} \cdot \left(\mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{L}^\mathrm{A} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{A}^\mathrm{I}}^*$$
$$= \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{L}^\mathrm{A} \cdot \mathbf{Q}_\mathrm{I}^\mathrm{I}\right) \cdot {\mathbf{Q}_\mathrm{A}^\mathrm{I}}^* = \mathbf{D}_\mathrm{A}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{A}\right) = \mathbf{D}_\mathrm{A}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{I}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{A} = \mathbf{D}_\mathrm{A}^\mathrm{I} \cdot \mathbf{D}_\mathrm{L}^\mathrm{A} = \mathbf{D}_\mathrm{L}^\mathrm{I} = \mathbf{a}_\mathrm{L}^\mathrm{I}$$
$$\mathbf{a}_\mathrm{B}^\mathrm{L} = \left(\mathbf{Q}_\mathrm{L}^\mathrm{I}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{B}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{L}^\mathrm{I}\right)^\mathrm{I} = \left(\mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{L}^\mathrm{I}\right) \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{L}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{B}^\mathrm{I}\right) = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{L}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{B}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{L} \cdot \mathbf{D}_\mathrm{B}^\mathrm{I} = \mathbf{D}_\mathrm{B}^\mathrm{L} = \mathbf{a}_\mathrm{B}^\mathrm{L}$$
$$\mathbf{a}_\mathrm{A}^\mathrm{L} = \left(\mathbf{Q}_\mathrm{L}^\mathrm{I}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{A}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{L}^\mathrm{I}\right)^\mathrm{I} = \left(\mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{L}^\mathrm{I}\right) \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{L}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{A}^\mathrm{I}\right) = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{L}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{A}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{L} \cdot \mathbf{D}_\mathrm{A}^\mathrm{I} = \mathbf{D}_\mathrm{A}^\mathrm{L} = \mathbf{a}_\mathrm{A}^\mathrm{L}$$
$$\mathbf{a}_\mathrm{I}^\mathrm{B} = \left(\mathbf{Q}_\mathrm{B}^\mathrm{I}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{I}\right)^\mathrm{I} = \left(\mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{Q}_\mathrm{B}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{B}^\mathrm{I}\right) \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{B}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{I}\right) = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{B}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{B} \cdot \mathbf{D}_\mathrm{I}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{B} = \mathbf{a}_\mathrm{I}^\mathrm{B}$$
$$\mathbf{a}_\mathrm{L}^\mathrm{B} = \left(\mathbf{Q}_\mathrm{B}^\mathrm{I}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{I}\right)^\mathrm{I} = \left(\mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{Q}_\mathrm{B}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{B}^\mathrm{I}\right) \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{B}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{I}\right) = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{B}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{B} \cdot \mathbf{D}_\mathrm{L}^\mathrm{I} = \mathbf{D}_\mathrm{L}^\mathrm{B} = \mathbf{a}_\mathrm{L}^\mathrm{B}$$
$$\mathbf{a}_\mathrm{I}^\mathrm{A} = \left(\mathbf{Q}_\mathrm{A}^\mathrm{I}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{A}^\mathrm{I}\right)^\mathrm{I} = \left(\mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{A}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{Q}_\mathrm{A}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{I}\right) \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{A}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{I}\right) = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{A}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{A} \cdot \mathbf{D}_\mathrm{I}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{A} = \mathbf{a}_\mathrm{I}^\mathrm{A}$$
$$\mathbf{a}_\mathrm{L}^\mathrm{A} = \left(\mathbf{Q}_\mathrm{A}^\mathrm{I}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \left(\mathbf{Q}_\mathrm{A}^\mathrm{I}\right)^\mathrm{I} = \left(\mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{A}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{Q}_\mathrm{A}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} \cdot \mathbf{Q}_\mathrm{A}^\mathrm{I}\right) \cdot \mathbf{Q}_\mathrm{I}^{\mathrm{I}^*} = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{A}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{I}\right) = \mathbf{D}_\mathrm{I}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{A}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{L}^\mathrm{I} = \mathbf{D}_\mathrm{I}^\mathrm{A} \cdot \mathbf{D}_\mathrm{L}^\mathrm{I} = \mathbf{D}_\mathrm{L}^\mathrm{A} = \mathbf{a}_\mathrm{L}^\mathrm{A}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_\mathrm{I}^\mathrm{L} = \left(\mathbf{Q}_\mathrm{B}^\mathrm{L}\right)^\mathrm{I} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{L}\right)^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^*\right)^* = \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} \cdot \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^*$$
$$= \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot \left({\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} \cdot \mathbf{Q}_\mathrm{L}^\mathrm{I}\right) \cdot {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^* = \mathbf{D}_\mathrm{B}^\mathrm{I} \cdot \left({\mathbf{D}_\mathrm{L}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B}\right) = \mathbf{D}_\mathrm{B}^\mathrm{I} \cdot {\mathbf{D}_\mathrm{L}^\mathrm{I}}^\mathrm{T} \cdot \mathbf{a}_\mathrm{I}^\mathrm{B} = \mathbf{D}_\mathrm{B}^\mathrm{I} \cdot \mathbf{D}_\mathrm{I}^\mathrm{L} \cdot \mathbf{D}_\mathrm{I}^\mathrm{B}$$
$$\mathbf{a}_\mathrm{L}^\mathrm{L} = \left(\mathbf{Q}_\mathrm{B}^\mathrm{L}\right)^\mathrm{I} \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{L}\right)^{\mathrm{I}^*} = \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} \cdot \left(\mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^*\right)^* = \mathbf{Q}_\mathrm{B}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{L}^\mathrm{I}}^* \cdot \mathbf{a}_\mathrm{L}^\mathrm{B} \cdot \mathbf{Q}_\mathrm{L}^\mathrm{I} \cdot {\mathbf{Q}_\mathrm{B}^\mathrm{I}}^*$$

$$\begin{aligned}
&= \mathbf{Q}_B^I \cdot \left({\mathbf{Q}_L^I}^* \cdot \mathbf{a}_L^B \cdot \mathbf{Q}_L^I\right) \cdot {\mathbf{Q}_B^I}^* = \mathbf{D}_B^I \cdot \left({\mathbf{D}_L^I}^T \cdot \mathbf{a}_L^B\right) = \mathbf{D}_B^I \cdot {\mathbf{D}_L^I}^T \cdot \mathbf{a}_L^B = \mathbf{D}_B^I \cdot \mathbf{D}_I^L \cdot \mathbf{D}_L^B \\
\mathbf{a}_I^L &= \left(\mathbf{Q}_A^L\right)^I \cdot \mathbf{a}_I^A \cdot \left(\mathbf{Q}_A^L\right)^{I^*} = \mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^* \cdot \mathbf{a}_I^A \cdot \left(\mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^*\right)^* = \mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^* \cdot \mathbf{a}_I^A \cdot \mathbf{Q}_L^I \cdot {\mathbf{Q}_A^I}^* \\
&= \mathbf{Q}_A^I \cdot \left({\mathbf{Q}_L^I}^* \cdot \mathbf{a}_I^A \cdot \mathbf{Q}_L^I\right) \cdot {\mathbf{Q}_A^I}^* = \mathbf{D}_A^I \cdot \left({\mathbf{D}_L^I}^T \cdot \mathbf{a}_I^A\right) = \mathbf{D}_A^I \cdot {\mathbf{D}_L^I}^T \cdot \mathbf{a}_I^A = \mathbf{D}_A^I \cdot \mathbf{D}_I^L \cdot \mathbf{D}_I^A \\
\mathbf{a}_L^L &= \left(\mathbf{Q}_A^L\right)^I \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^L\right)^{I^*} = \mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^* \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^*\right)^* = \mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^* \cdot \mathbf{a}_L^A \cdot \mathbf{Q}_L^I \cdot {\mathbf{Q}_A^I}^* \\
&= \mathbf{Q}_A^I \cdot \left({\mathbf{Q}_L^I}^* \cdot \mathbf{a}_L^A \cdot \mathbf{Q}_L^I\right) \cdot {\mathbf{Q}_A^I}^* = \mathbf{D}_A^I \cdot \left({\mathbf{D}_L^I}^T \cdot \mathbf{a}_L^A\right) = \mathbf{D}_A^I \cdot {\mathbf{D}_L^I}^T \cdot \mathbf{a}_L^A = \mathbf{D}_A^I \cdot \mathbf{D}_I^L \cdot \mathbf{D}_L^A \\
\mathbf{a}_I^B &= \left(\mathbf{Q}_A^B\right)^I \cdot \mathbf{a}_I^A \cdot \left(\mathbf{Q}_A^B\right)^{I^*} = \mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^* \cdot \mathbf{a}_I^A \cdot \left(\mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^*\right)^* = \mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^* \cdot \mathbf{a}_I^A \cdot \mathbf{Q}_B^I \cdot {\mathbf{Q}_A^I}^* \\
&= \mathbf{Q}_A^I \cdot \left({\mathbf{Q}_B^I}^* \cdot \mathbf{a}_I^A \cdot \mathbf{Q}_B^I\right) \cdot {\mathbf{Q}_A^I}^* = \mathbf{D}_A^I \cdot \left({\mathbf{D}_B^I}^T \cdot \mathbf{a}_I^A\right) = \mathbf{D}_A^I \cdot {\mathbf{D}_B^I}^T \cdot \mathbf{a}_I^A = \mathbf{D}_A^I \cdot \mathbf{D}_I^B \cdot \mathbf{D}_I^A \\
\mathbf{a}_L^B &= \left(\mathbf{Q}_A^B\right)^I \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^B\right)^{I^*} = \mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^* \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^*\right)^* = \mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^* \cdot \mathbf{a}_L^A \cdot \mathbf{Q}_B^I \cdot {\mathbf{Q}_A^I}^* \\
&= \mathbf{Q}_A^I \cdot \left({\mathbf{Q}_B^I}^* \cdot \mathbf{a}_L^A \cdot \mathbf{Q}_B^I\right) \cdot {\mathbf{Q}_A^I}^* = \mathbf{D}_A^I \cdot \left({\mathbf{D}_B^I}^T \cdot \mathbf{a}_L^A\right) = \mathbf{D}_A^I \cdot {\mathbf{D}_B^I}^T \cdot \mathbf{a}_L^A = \mathbf{D}_A^I \cdot \mathbf{D}_I^B \cdot \mathbf{D}_L^A \\
\mathbf{a}_L^B &= \left(\mathbf{Q}_A^B\right)^L \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^B\right)^{L^*} = \mathbf{Q}_A^L \cdot {\mathbf{Q}_B^L}^* \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^L \cdot {\mathbf{Q}_B^L}^*\right)^* = \mathbf{Q}_A^L \cdot {\mathbf{Q}_B^L}^* \cdot \mathbf{a}_L^A \cdot \mathbf{Q}_B^L \cdot {\mathbf{Q}_A^L}^* \\
&= \mathbf{Q}_A^L \cdot \left({\mathbf{Q}_B^L}^* \cdot \mathbf{a}_L^A \cdot \mathbf{Q}_B^L\right) \cdot {\mathbf{Q}_A^L}^* = \mathbf{D}_A^L \cdot \left({\mathbf{D}_B^L}^T \cdot \mathbf{a}_L^A\right) = \mathbf{D}_A^L \cdot {\mathbf{D}_B^L}^T \cdot \mathbf{a}_L^A = \mathbf{D}_A^L \cdot \mathbf{D}_L^B \cdot \mathbf{D}_L^A \\
\mathbf{a}_I^B &= \left(\mathbf{Q}_B^L\right)^{I^*} \cdot \mathbf{a}_I^L \cdot \left(\mathbf{Q}_B^L\right)^I = \left(\mathbf{Q}_B^I \cdot {\mathbf{Q}_L^I}^*\right)^* \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_B^I \cdot {\mathbf{Q}_L^I}^* = \mathbf{Q}_L^I \cdot {\mathbf{Q}_B^I}^* \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_B^I \cdot {\mathbf{Q}_L^I}^* \\
&= \mathbf{Q}_L^I \cdot \left({\mathbf{Q}_B^I}^* \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_B^I\right) \cdot {\mathbf{Q}_L^I}^* = \mathbf{D}_L^I \cdot \left({\mathbf{D}_B^I}^T \cdot \mathbf{a}_I^L\right) = \mathbf{D}_L^I \cdot {\mathbf{D}_B^I}^T \cdot \mathbf{a}_I^L = \mathbf{D}_L^I \cdot \mathbf{D}_I^B \cdot \mathbf{D}_I^L \\
\mathbf{a}_L^B &= \left(\mathbf{Q}_B^L\right)^{I^*} \cdot \mathbf{a}_L^L \cdot \left(\mathbf{Q}_B^L\right)^I = \left(\mathbf{Q}_B^I \cdot {\mathbf{Q}_L^I}^*\right)^* \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_B^I \cdot {\mathbf{Q}_L^I}^* = \mathbf{Q}_L^I \cdot {\mathbf{Q}_B^I}^* \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_B^I \cdot {\mathbf{Q}_L^I}^* \\
&= \mathbf{Q}_L^I \cdot \left({\mathbf{Q}_B^I}^* \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_B^I\right) \cdot {\mathbf{Q}_L^I}^* = \mathbf{D}_L^I \cdot \left({\mathbf{D}_B^I}^T \cdot \mathbf{a}_L^L\right) = \mathbf{D}_L^I \cdot {\mathbf{D}_B^I}^T \cdot \mathbf{a}_L^L = \mathbf{D}_L^I \cdot \mathbf{D}_I^B \cdot \mathbf{D}_L^L = \mathbf{D}_L^I \cdot \mathbf{D}_I^B \\
\mathbf{a}_I^A &= \left(\mathbf{Q}_A^L\right)^{I^*} \cdot \mathbf{a}_I^L \cdot \left(\mathbf{Q}_A^L\right)^I = \left(\mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^*\right)^* \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^* = \mathbf{Q}_L^I \cdot {\mathbf{Q}_A^I}^* \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^* \\
&= \mathbf{Q}_L^I \cdot \left({\mathbf{Q}_A^I}^* \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_A^I\right) \cdot {\mathbf{Q}_L^I}^* = \mathbf{D}_L^I \cdot \left({\mathbf{D}_A^I}^T \cdot \mathbf{a}_I^L\right) = \mathbf{D}_L^I \cdot {\mathbf{D}_A^I}^T \cdot \mathbf{a}_I^L = \mathbf{D}_L^I \cdot \mathbf{D}_I^A \cdot \mathbf{D}_I^L \\
\mathbf{a}_L^A &= \left(\mathbf{Q}_A^L\right)^{I^*} \cdot \mathbf{a}_L^L \cdot \left(\mathbf{Q}_A^L\right)^I = \left(\mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^*\right)^* \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^* = \mathbf{Q}_L^I \cdot {\mathbf{Q}_A^I}^* \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_A^I \cdot {\mathbf{Q}_L^I}^* \\
&= \mathbf{Q}_L^I \cdot \left({\mathbf{Q}_A^I}^* \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_A^I\right) \cdot {\mathbf{Q}_L^I}^* = \mathbf{D}_L^I \cdot \left({\mathbf{D}_A^I}^T \cdot \mathbf{a}_L^L\right) = \mathbf{D}_L^I \cdot {\mathbf{D}_A^I}^T \cdot \mathbf{a}_L^L = \mathbf{D}_L^I \cdot \mathbf{D}_I^A \cdot \mathbf{D}_L^L = \mathbf{D}_L^I \cdot \mathbf{D}_I^A \\
\mathbf{a}_I^A &= \left(\mathbf{Q}_A^B\right)^{I^*} \cdot \mathbf{a}_I^B \cdot \left(\mathbf{Q}_A^B\right)^I = \left(\mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^*\right)^* \cdot \mathbf{a}_I^B \cdot \mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^* = \mathbf{Q}_B^I \cdot {\mathbf{Q}_A^I}^* \cdot \mathbf{a}_I^B \cdot \mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^* \\
&= \mathbf{Q}_B^I \cdot \left({\mathbf{Q}_A^I}^* \cdot \mathbf{a}_I^B \cdot \mathbf{Q}_A^I\right) \cdot {\mathbf{Q}_B^I}^* = \mathbf{D}_B^I \cdot \left({\mathbf{D}_A^I}^T \cdot \mathbf{a}_I^B\right) = \mathbf{D}_B^I \cdot {\mathbf{D}_A^I}^T \cdot \mathbf{a}_I^B = \mathbf{D}_B^I \cdot \mathbf{D}_I^A \cdot \mathbf{D}_I^B \\
\mathbf{a}_L^A &= \left(\mathbf{Q}_A^B\right)^{I^*} \cdot \mathbf{a}_L^B \cdot \left(\mathbf{Q}_A^B\right)^I = \left(\mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^*\right)^* \cdot \mathbf{a}_L^B \cdot \mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^* = \mathbf{Q}_B^I \cdot {\mathbf{Q}_A^I}^* \cdot \mathbf{a}_L^B \cdot \mathbf{Q}_A^I \cdot {\mathbf{Q}_B^I}^* \\
&= \mathbf{Q}_B^I \cdot \left({\mathbf{Q}_A^I}^* \cdot \mathbf{a}_L^B \cdot \mathbf{Q}_A^I\right) \cdot {\mathbf{Q}_B^I}^* = \mathbf{D}_B^I \cdot \left({\mathbf{D}_A^I}^T \cdot \mathbf{a}_L^B\right) = \mathbf{D}_B^I \cdot {\mathbf{D}_A^I}^T \cdot \mathbf{a}_L^B = \mathbf{D}_B^I \cdot \mathbf{D}_I^A \cdot \mathbf{D}_L^B \\
\mathbf{a}_L^A &= \left(\mathbf{Q}_A^B\right)^{L^*} \cdot \mathbf{a}_L^B \cdot \left(\mathbf{Q}_A^B\right)^L = \left(\mathbf{Q}_A^L \cdot {\mathbf{Q}_B^L}^*\right)^* \cdot \mathbf{a}_L^B \cdot \mathbf{Q}_A^L \cdot {\mathbf{Q}_B^L}^* = \mathbf{Q}_B^L \cdot {\mathbf{Q}_A^L}^* \cdot \mathbf{a}_L^B \cdot \mathbf{Q}_A^L \cdot {\mathbf{Q}_B^L}^* \\
&= \mathbf{Q}_B^L \cdot \left({\mathbf{Q}_A^L}^* \cdot \mathbf{a}_L^B \cdot \mathbf{Q}_A^L\right) \cdot {\mathbf{Q}_B^L}^* = \mathbf{D}_B^L \cdot \left({\mathbf{D}_A^L}^T \cdot \mathbf{a}_L^B\right) = \mathbf{D}_B^L \cdot {\mathbf{D}_A^L}^T \cdot \mathbf{a}_L^B = \mathbf{D}_B^L \cdot \mathbf{D}_L^A \cdot \mathbf{D}_L^B
\end{aligned}$$

# 6 まとめ

## 6.1 座標系の回転

方向余弦マトリクスは基準座標系の如何に関わらず利用できる。他の体(非可換体)においては、それらを表現する基準の座標系と演算対象の座標軸ベクトルを表現する基準の座標系が一致していれば利用できる。

$$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \Rightarrow \left(\underline{\mathrm{x}}_{\mathrm{B}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{B}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{B}}^{\mathrm{I}}\right) = \left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \quad \text{:方向余弦マトリクスによる座標系の回転}$$

$$\left.\begin{array}{l} \underline{\mathrm{x}}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \underline{\mathrm{x}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \\ \underline{\mathrm{y}}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \underline{\mathrm{y}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \\ \underline{\mathrm{z}}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \underline{\mathrm{z}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \end{array}\right\} \text{:オイラー角型四元数による座標系の回転}$$

$$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \Rightarrow \left(\underline{\mathrm{x}}_{\mathrm{B}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{B}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{B}}^{\mathrm{I}}\right) = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{I}}\right) \quad \text{:固定角型回転マトリクスよる座標系の回転}$$

$$\left.\begin{array}{l} \underline{\mathrm{x}}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \underline{\mathrm{x}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \\ \underline{\mathrm{y}}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \underline{\mathrm{y}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \\ \underline{\mathrm{z}}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \underline{\mathrm{z}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \end{array}\right\} \text{:固定角型四元数による座標系の回転}$$

## 6.2 ベクトルの回転

4種の体(非可換体)はいずれも基準座標系が一致していれば利用できる。

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \quad \text{:方向余弦マトリクスによるベクトルの回転}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \quad \text{:オイラー角型四元数によるベクトルの回転}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \quad \text{:固定角型回転マトリクスによるベクトルの回転}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \quad \text{:固定角型四元数によるベクトルの回転}$$

## 6.3 ベクトルの座標変換

方向余弦マトリクスおよびオイラー角型四元数は、基準座標系の如何に関わらず利用できる。

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \Rightarrow \begin{pmatrix} \mathrm{a}_{\mathrm{x_B}}^{\mathrm{I}} \\ \mathrm{a}_{\mathrm{y_B}}^{\mathrm{I}} \\ \mathrm{a}_{\mathrm{z_B}}^{\mathrm{I}} \end{pmatrix} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \begin{pmatrix} \mathrm{a}_{\mathrm{x_B}}^{\mathrm{L}} \\ \mathrm{a}_{\mathrm{y_B}}^{\mathrm{L}} \\ \mathrm{a}_{\mathrm{z_B}}^{\mathrm{L}} \end{pmatrix} \quad \text{:方向余弦マトリクスによるベクトルの座標変換}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \otimes \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \Rightarrow \begin{pmatrix} \mathrm{a}_{\mathrm{x_B}}^{\mathrm{I}} \\ \mathrm{a}_{\mathrm{y_B}}^{\mathrm{I}} \\ \mathrm{a}_{\mathrm{z_B}}^{\mathrm{I}} \end{pmatrix} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \begin{pmatrix} 0 \\ \mathrm{a}_{\mathrm{x_B}}^{\mathrm{L}} \\ \mathrm{a}_{\mathrm{y_B}}^{\mathrm{L}} \\ \mathrm{a}_{\mathrm{z_B}}^{\mathrm{L}} \end{pmatrix} \otimes \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \quad \text{:オイラー角型四元数によるベクトルの座標変換}$$

固定角型回転マトリクスおよび固定角型四元数は、それらを方向余弦マトリクスに変形し、さらにベクトルを方向余弦マトリクスと見なして一つの方向余弦マトリクスに合成できるならば、座標変換に利用できる。この体(非可換体)は座標変換に不向きであり、わざわざ利用するメリットは無い。

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}}\right) \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} =\cdot\, \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$

$$\Rightarrow \begin{pmatrix} \mathrm{a}_{\mathrm{x_B}}^{\mathrm{I}} \\ \mathrm{a}_{\mathrm{y_B}}^{\mathrm{I}} \\ \mathrm{a}_{\mathrm{z_B}}^{\mathrm{I}} \end{pmatrix} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \begin{pmatrix} \mathrm{a}_{\mathrm{x_B}}^{\mathrm{L}} \\ \mathrm{a}_{\mathrm{y_B}}^{\mathrm{L}} \\ \mathrm{a}_{\mathrm{z_B}}^{\mathrm{L}} \end{pmatrix} \quad \text{:固定角型回転マトリクスによるベクトルの座標変換}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \otimes \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \otimes \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right) \otimes \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \otimes \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right) \otimes \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \otimes \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right) = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \otimes \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$

$$= \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$

$$\Rightarrow \begin{pmatrix} \mathrm{a}_{\mathrm{x_B}}^{\mathrm{I}} \\ \mathrm{a}_{\mathrm{y_B}}^{\mathrm{I}} \\ \mathrm{a}_{\mathrm{z_B}}^{\mathrm{I}} \end{pmatrix} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \otimes \begin{pmatrix} 0 \\ \mathrm{a}_{\mathrm{x_B}}^{\mathrm{L}} \\ \mathrm{a}_{\mathrm{y_B}}^{\mathrm{L}} \\ \mathrm{a}_{\mathrm{z_B}}^{\mathrm{L}} \end{pmatrix} \otimes \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} : \text{固定角型四元数によるベクトルの座標変換}$$