表5-2　ベクトルの回転

| No. | 方向余弦マトリクスによる回転 | | | | 固定角型回転マトリクスによる回転 | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | $\mathbf{a}_L^I = \mathbf{D}_L^I \cdot \mathbf{a}_I^I$ | ◎ | $\mathbf{a}_I^I = \mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^I$ | ◎ | $\mathbf{a}_L^I = (\mathbf{D}_L^I)^I \cdot \mathbf{a}_I^I$ | ◎ | $\mathbf{a}_I^I = (\mathbf{D}_L^I)^{I\,T} \cdot \mathbf{a}_L^I$ | ◎ |
| 2 | $\mathbf{a}_L^B = \mathbf{D}_L^I \cdot \mathbf{a}_I^B$ | × | $\mathbf{a}_I^A = \mathbf{D}_L^{I\,T} \cdot \mathbf{a}_L^A$ | × | $\mathbf{a}_L^B = (\mathbf{D}_L^I)^I \cdot \mathbf{a}_I^B$ | × | $\mathbf{a}_I^A = (\mathbf{D}_L^I)^{I\,T} \cdot \mathbf{a}_L^A$ | × |
| 3 | $\mathbf{a}_B^I = \mathbf{D}_B^L \cdot \mathrm{a}_L^I$ | × | $\mathbf{a}_L^I = \mathbf{D}_B^{L\,T} \cdot \mathbf{a}_B^I$ | × | $\mathbf{a}_B^I = (\mathbf{D}_B^L)^I \cdot \mathbf{a}_L^I$ | ◎ | $\mathbf{a}_L^I = (\mathbf{D}_B^L)^{I\,T} \cdot \mathbf{a}_B^I$ | ◎ |
| 4 | $\mathbf{a}_B^L = \mathbf{D}_B^L \cdot \mathbf{a}_L^L$ | ◎ | $\mathbf{a}_L^L = \mathbf{D}_B^{L\,T} \cdot \mathbf{a}_B^L$ | ◎ | $\mathbf{a}_B^L = (\mathbf{D}_B^L)^I \cdot \mathbf{a}_L^L$ | × | $\mathbf{a}_L^L = (\mathbf{D}_B^L)^{I\,T} \cdot \mathbf{a}_B^L$ | × |
| 5 | $\mathbf{a}_A^I = \mathbf{D}_A^L \cdot \mathbf{a}_L^I$ | × | $\mathbf{a}_L^I = \mathbf{D}_A^{L\,T} \cdot \mathbf{a}_A^I$ | × | $\mathbf{a}_A^I = (\mathbf{D}_A^L)^I \cdot \mathbf{a}_L^I$ | ◎ | $\mathbf{a}_L^I = (\mathbf{D}_A^L)^{I\,T} \cdot \mathbf{a}_A^I$ | ◎ |
| 6 | $\mathbf{a}_A^L = \mathbf{D}_A^L \cdot \mathbf{a}_L^L$ | ◎ | $\mathbf{a}_L^L = \mathbf{D}_A^{L\,T} \cdot \mathbf{a}_A^L$ | ◎ | $\mathbf{a}_A^L = (\mathbf{D}_A^L)^I \cdot \mathbf{a}_L^L$ | × | $\mathbf{a}_L^L = (\mathbf{D}_A^L)^{I\,T} \cdot \mathbf{a}_A^L$ | × |
| 7 | $\mathbf{a}_B^I = \mathbf{D}_B^I \cdot \mathbf{a}_I^I$ | ◎ | $\mathbf{a}_I^I = \mathbf{D}_B^{I\,T} \cdot \mathbf{a}_B^I$ | ◎ | $\mathbf{a}_B^I = (\mathbf{D}_B^I)^I \cdot \mathbf{a}_I^I$ | ◎ | $\mathbf{a}_I^I = (\mathbf{D}_B^I)^{I\,T} \cdot \mathbf{a}_B^I$ | ◎ |
| 8 | $\mathbf{a}_B^L = \mathbf{D}_B^I \cdot \mathbf{a}_I^L$ | × | $\mathbf{a}_I^L = \mathbf{D}_B^{I\,T} \cdot \mathbf{a}_B^L$ | × | $\mathbf{a}_B^L = (\mathbf{D}_B^I)^I \cdot \mathbf{a}_I^L$ | × | $\mathbf{a}_I^L = (\mathbf{D}_B^I)^{I\,T} \cdot \mathbf{a}_B^L$ | × |
| 9 | $\mathbf{a}_A^I = \mathbf{D}_A^I \cdot \mathbf{a}_I^I$ | ◎ | $\mathbf{a}_I^I = \mathbf{D}_A^{I\,T} \cdot \mathbf{a}_A^I$ | ◎ | $\mathbf{a}_A^I = (\mathbf{D}_A^I)^I \cdot \mathbf{a}_I^I$ | ◎ | $\mathbf{a}_I^I = (\mathbf{D}_A^I)^{I\,T} \cdot \mathbf{a}_A^I$ | ◎ |
| 10 | $\mathbf{a}_A^L = \mathbf{D}_A^I \cdot \mathbf{a}_I^L$ | × | $\mathbf{a}_I^L = \mathbf{D}_A^{I\,T} \cdot \mathbf{a}_A^L$ | × | $\mathbf{a}_A^L = (\mathbf{D}_A^I)^I \cdot \mathbf{a}_I^L$ | × | $\mathbf{a}_I^L = (\mathbf{D}_A^I)^{I\,T} \cdot \mathbf{a}_A^L$ | × |
| 11 | $\mathbf{a}_A^I = \mathbf{D}_A^B \cdot \mathbf{a}_B^I$ | × | $\mathbf{a}_B^I = \mathbf{D}_A^{B\,T} \cdot \mathbf{a}_A^I$ | × | $\mathbf{a}_A^I = (\mathbf{D}_A^B)^I \cdot \mathbf{a}_B^I$ | ◎ | $\mathbf{a}_B^I = (\mathbf{D}_A^B)^{I\,T} \cdot \mathbf{a}_A^I$ | ◎ |
| 12 | $\mathbf{a}_A^L = \mathbf{D}_A^B \cdot \mathbf{a}_B^L$ | × | $\mathbf{a}_B^L = \mathbf{D}_A^{B\,T} \cdot \mathbf{a}_A^L$ | × | $\mathbf{a}_A^L = (\mathbf{D}_A^B)^I \cdot \mathbf{a}_B^L$ | × | $\mathbf{a}_B^L = (\mathbf{D}_A^B)^{I\,T} \cdot \mathbf{a}_A^L$ | × |
| 13 | | | | | $\mathbf{a}_A^L = (\mathbf{D}_A^B)^L \cdot \mathbf{a}_B^L$ | ◎ | $\mathbf{a}_B^L = (\mathbf{D}_A^B)^{L\,T} \cdot \mathbf{a}_A^L$ | ◎ |

| No. | オイラー角型四元数による回転 | | | | 固定角型四元数による回転 | | | |
|---|---|---|---|---|---|---|---|---|
| 21 | $\mathbf{a}_L^I = \mathbf{Q}_L^I \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_L^{I\,*}$ | ◎ | $\mathbf{a}_I^I = \mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_L^I$ | ◎ | $\mathbf{a}_L^I = (\mathbf{Q}_L^I)^I \cdot \mathbf{a}_I^I \cdot (\mathbf{Q}_L^I)^{I*}$ | ◎ | $\mathbf{a}_I^I = (\mathbf{Q}_L^I)^{I*} \cdot \mathbf{a}_L^I \cdot (\mathbf{Q}_L^I)^I$ | ◎ |
| 22 | $\mathbf{a}_L^B = \mathbf{Q}_L^I \cdot \mathbf{a}_I^B \cdot \mathbf{Q}_L^{I\,*}$ | × | $\mathbf{a}_I^A = \mathbf{Q}_L^{I\,*} \cdot \mathbf{a}_L^A \cdot \mathbf{Q}_L^I$ | × | $\mathbf{a}_L^B = (\mathbf{Q}_L^I)^I \cdot \mathbf{a}_I^B \cdot (\mathbf{Q}_L^I)^{I*}$ | × | $\mathbf{a}_I^A = (\mathbf{Q}_L^I)^{I*} \cdot \mathbf{a}_L^A \cdot (\mathbf{Q}_L^I)^I$ | × |
| 23 | $\mathbf{a}_B^I = \mathbf{Q}_B^L \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_B^{L\,*}$ | × | $\mathbf{a}_L^I = \mathbf{Q}_B^{L\,*} \cdot \mathbf{a}_B^I \cdot \mathbf{Q}_B^L$ | × | $\mathbf{a}_B^I = (\mathbf{Q}_B^L)^I \cdot \mathbf{a}_L^I \cdot (\mathbf{Q}_B^L)^{I*}$ | ◎ | $\mathbf{a}_L^I = (\mathbf{Q}_B^L)^{I*} \cdot \mathbf{a}_B^I \cdot (\mathbf{Q}_B^L)^I$ | ◎ |
| 24 | $\mathbf{a}_B^L = \mathbf{Q}_B^L \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_B^{L\,*}$ | ◎ | $\mathbf{a}_L^L = \mathbf{Q}_B^{L\,*} \cdot \mathbf{a}_B^L \cdot \mathbf{Q}_B^L$ | ◎ | $\mathbf{a}_B^L = (\mathbf{Q}_B^L)^I \cdot \mathbf{a}_L^L \cdot (\mathbf{Q}_B^L)^{I*}$ | × | $\mathbf{a}_L^L = (\mathbf{Q}_B^L)^{I*} \cdot \mathbf{a}_B^L \cdot (\mathbf{Q}_B^L)^I$ | × |
| 25 | $\mathbf{a}_A^I = \mathbf{Q}_A^L \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_A^{L\,*}$ | × | $\mathbf{a}_L^I = \mathbf{Q}_A^{L\,*} \cdot \mathbf{a}_A^I \cdot \mathbf{Q}_A^L$ | × | $\mathbf{a}_A^I = (\mathbf{Q}_A^L)^I \cdot \mathbf{a}_L^I \cdot (\mathbf{Q}_A^L)^{I*}$ | ◎ | $\mathbf{a}_L^I = (\mathbf{Q}_A^L)^{I*} \cdot \mathbf{a}_A^I \cdot (\mathbf{Q}_A^L)^I$ | ◎ |
| 26 | $\mathbf{a}_A^L = \mathbf{Q}_A^L \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_A^{L\,*}$ | ◎ | $\mathbf{a}_L^L = \mathbf{Q}_A^{L\,*} \cdot \mathbf{a}_A^L \cdot \mathbf{Q}_A^L$ | ◎ | $\mathbf{a}_A^L = (\mathbf{Q}_A^L)^I \cdot \mathbf{a}_L^L \cdot (\mathbf{Q}_A^L)^{I*}$ | × | $\mathbf{a}_L^L = (\mathbf{Q}_A^L)^{I*} \cdot \mathbf{a}_A^L \cdot (\mathbf{Q}_A^L)^I$ | × |
| 27 | $\mathbf{a}_B^I = \mathbf{Q}_B^I \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_B^{I\,*}$ | ◎ | $\mathbf{a}_I^I = \mathbf{Q}_B^{I\,*} \cdot \mathbf{a}_B^I \cdot \mathbf{Q}_B^I$ | ◎ | $\mathbf{a}_B^I = (\mathbf{Q}_B^I)^I \cdot \mathbf{a}_I^I \cdot (\mathbf{Q}_B^I)^{I*}$ | ◎ | $\mathbf{a}_I^I = (\mathbf{Q}_B^I)^{I*} \cdot \mathbf{a}_B^I \cdot (\mathbf{Q}_B^I)^I$ | ◎ |
| 28 | $\mathbf{a}_B^L = \mathbf{Q}_B^I \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_B^{I\,*}$ | × | $\mathbf{a}_I^L = \mathbf{Q}_B^{I\,*} \cdot \mathbf{a}_B^L \cdot \mathbf{Q}_B^I$ | × | $\mathbf{a}_B^L = (\mathbf{Q}_B^I)^I \cdot \mathbf{a}_I^L \cdot (\mathbf{Q}_B^I)^{I*}$ | × | $\mathbf{a}_I^L = (\mathbf{Q}_B^I)^{I*} \cdot \mathbf{a}_B^L \cdot (\mathbf{Q}_B^I)^I$ | × |
| 29 | $\mathbf{a}_A^I = \mathbf{Q}_A^I \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_A^{I\,*}$ | ◎ | $\mathbf{a}_I^I = \mathbf{Q}_A^{I\,*} \cdot \mathbf{a}_A^I \cdot \mathbf{Q}_A^I$ | ◎ | $\mathbf{a}_A^I = (\mathbf{Q}_A^I)^I \cdot \mathbf{a}_I^I \cdot (\mathbf{Q}_A^I)^{I*}$ | ◎ | $\mathbf{a}_I^I = (\mathbf{Q}_A^I)^{I*} \cdot \mathbf{a}_A^I \cdot (\mathbf{Q}_A^I)^I$ | ◎ |
| 30 | $\mathbf{a}_A^L = \mathbf{Q}_A^I \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_A^{I\,*}$ | × | $\mathbf{a}_I^L = \mathbf{Q}_A^{I\,*} \cdot \mathbf{a}_A^L \cdot \mathbf{Q}_A^I$ | × | $\mathbf{a}_A^L = (\mathbf{Q}_A^I)^I \cdot \mathbf{a}_I^L \cdot (\mathbf{Q}_A^I)^{I*}$ | × | $\mathbf{a}_I^L = (\mathbf{Q}_A^I)^{I*} \cdot \mathbf{a}_A^L \cdot (\mathbf{Q}_A^I)^I$ | × |
| 31 | $\mathbf{a}_A^I = \mathbf{Q}_A^B \cdot \mathbf{a}_B^I \cdot \mathbf{Q}_A^{B*}$ | × | $\mathbf{a}_B^I = \mathbf{Q}_A^{B*} \cdot \mathbf{a}_A^I \cdot \mathbf{Q}_A^B$ | × | $\mathbf{a}_A^I = (\mathbf{Q}_A^B)^I \cdot \mathbf{a}_B^I \cdot (\mathbf{Q}_A^B)^{I*}$ | ◎ | $\mathbf{a}_B^I = (\mathbf{Q}_A^B)^{I*} \cdot \mathbf{a}_A^I \cdot (\mathbf{Q}_A^B)^I$ | ◎ |
| 32 | $\mathbf{a}_A^L = \mathbf{Q}_A^B \cdot \mathbf{a}_B^L \cdot \mathbf{Q}_A^{B*}$ | × | $\mathbf{a}_B^L = \mathbf{Q}_A^{B*} \cdot \mathbf{a}_A^L \cdot \mathbf{Q}_A^B$ | × | $\mathbf{a}_A^L = (\mathbf{Q}_A^B)^I \cdot \mathbf{a}_B^L \cdot (\mathbf{Q}_A^B)^{I*}$ | × | $\mathbf{a}_B^L = (\mathbf{Q}_A^B)^{I*} \cdot \mathbf{a}_A^L \cdot (\mathbf{Q}_A^B)^I$ | × |
| 33 | | | | | $\mathbf{a}_A^L = (\mathbf{Q}_A^B)^L \cdot \mathbf{a}_B^L \cdot (\mathbf{Q}_A^B)^{L*}$ | ◎ | $\mathbf{a}_B^L = (\mathbf{Q}_A^B)^{L*} \cdot \mathbf{a}_A^L \cdot (\mathbf{Q}_A^B)^L$ | ◎ |

ベクトルの回転は固定角型回転マトリクスおよび固定角型四元数が利用できるが、基準の座標系が一致していなければならない。

任意のベクトルの回転は次のように、「慣性航法のアルゴリズム」に示した式(1.4-6)で表される。

$$\mathbf{b}^R = \left(\mathbf{D}_G^A\right)^R \cdot \mathbf{a}^R$$ :ベクトルの回転

$$\mathbf{a}_G^R = \left(\mathbf{D}_G^A\right)^R \cdot \mathbf{a}_A^R$$ :座標系Rで表した座標系Aの軸ベクトルを座標系Gへ回転

$\mathbf{a}^{\mathrm{R}}, \mathbf{b}^{\mathrm{R}}$ ：座標系Rで表したベクトル

$\left(\mathbf{D}_{\mathrm{G}}^{\mathrm{A}}\right)^{\mathrm{R}} = \mathbf{D}_{\mathrm{G}}^{\mathrm{R}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{R}^{\mathrm{T}}}$ ：座標系で表した座標系Aから座標系Gへの固定角型回転マトリクス

$\mathbf{D}_{\mathrm{G}}^{\mathrm{R}} = [\theta_1]_{\mathrm{i}} \cdot [\theta_2]_{\mathrm{j}} \cdots [\theta_{\mathrm{n}}]_{\mathrm{k}}$ ：座標系RからG（座標系でなくても良い）への回転を表すオイラー角

$\mathbf{D}_{\mathrm{A}}^{\mathrm{R}} = [\varphi_1]_{\mathrm{e}} \cdot [\varphi_2]_{\mathrm{f}} \cdots [\varphi_{\mathrm{m}}]_{\mathrm{g}}$ ：座標系RからA（座標系でなくても良い）への回転を表すオイラー角

方向余弦マトリクスは右上添え字が示す座標系を基準座標系とした固定角型回転マトリクスと等しいので、方向余弦マトリクスの右上添え字側から右下添え字側に向かって基準座標系が同じベクトルを回転させることができる。

$$\mathbf{D}_{\mathbf{G}}^{\mathbf{R}} = \mathbf{D}_{\mathrm{G}}^{\mathrm{R}} \cdot \mathbf{D}_{\mathrm{R}}^{\mathrm{R}^{\mathrm{T}}} = \left(\mathbf{D}_{\mathrm{G}}^{\mathrm{R}}\right)^{\mathrm{R}}$$

$\mathbf{b}^{\mathrm{R}} = \mathbf{D}_{\mathrm{G}}^{\mathrm{R}} \cdot \mathbf{a}^{\mathrm{R}}$ ：座標系Rで表したベクトルのRからGへの回転

(1) 方向余弦マトリクスによるベクトルの回転

$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$

この方向余弦マトリクスは座標系Iで表された任意のベクトルに対し、座標系Iにおいて座標系Iから座標系Lへの回転と同じ回転を与える。ここでは回転の量が分かるように座標軸ベクトルを対象としたので右下添え字に座標系の記号を示しているが、基準の座標系が一致する任意のベクトルが回転できる。

$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$

$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$

$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$

$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$

$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$

$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$

$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$

$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$

$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$

基準の座標系が一致していないので変換できない例。

$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$

$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$

$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$

$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$

$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$

$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{B}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$

$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{B}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$

$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$

$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$

$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

(2) 固定角型回転マトリクスによるベクトルの回転

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

(3) オイラー角型四元数によるベクトルの回転

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

(4) 固定角型四元数によるベクトルの回転

$$\begin{aligned}\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} &= \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \\ &= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}\end{aligned}$$
$$\begin{aligned}\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} &= \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^{*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \\ &= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}\end{aligned}$$
$$\begin{aligned}\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} &= \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^{*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \\ &= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\end{aligned}$$
$$\begin{aligned}\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} &= \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \\ &= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}\end{aligned}$$
$$\begin{aligned}\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} &= \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \\ &= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\end{aligned}$$
$$\begin{aligned}\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} &= \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}\right)^{*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \\ &= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\end{aligned}$$

$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*}\right)^* \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\m

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^{*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}\right)^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{A}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

表5-3　ベクトルの座標変換

| No. | 方向余弦マトリクスによる座標変換 | | 固定角型回転マトリクスによる座標変換 | |
|---|---|---|---|---|
| 1 | $\mathbf{a}_B^I = \mathbf{D}_L^I \cdot \mathbf{a}_B^L$ ◎ | $\mathbf{a}_B^L = {\mathbf{D}_L^I}^T \cdot \mathbf{a}_B^I$ ◎ | $\mathbf{a}_B^I = \left(\mathbf{D}_L^I\right)^I \cdot \mathbf{a}_B^L$ ◎ | $\mathbf{a}_B^L = \left(\mathbf{D}_L^I\right)^{I^T} \cdot \mathbf{a}_B^I$ ◎ |
| 2 | $\mathbf{a}_A^I = \mathbf{D}_L^I \cdot \mathbf{a}_A^L$ ◎ | $\mathbf{a}_A^L = {\mathbf{D}_L^I}^T \cdot \mathbf{a}_A^I$ ◎ | $\mathbf{a}_A^I = \left(\mathbf{D}_L^I\right)^I \cdot \mathbf{a}_A^L$ ◎ | $\mathbf{a}_A^L = \left(\mathbf{D}_L^I\right)^{I^T} \cdot \mathbf{a}_A^I$ ◎ |
| 3 | $\mathbf{a}_I^L = \mathbf{D}_B^L \cdot \mathbf{a}_I^B$ ◎ | $\mathbf{a}_I^B = {\mathbf{D}_B^L}^T \cdot \mathbf{a}_I^L$ ◎ | $\mathbf{a}_I^L = \left(\mathbf{D}_B^L\right)^I \cdot \mathbf{a}_I^B$ × | $\mathbf{a}_I^B = \left(\mathbf{D}_B^L\right)^{I^T} \cdot \mathbf{a}_I^L$ × |
| 4 | $\mathbf{a}_L^L = \mathbf{D}_B^L \cdot \mathbf{a}_L^B$ ◎ | $\mathbf{a}_L^B = {\mathbf{D}_B^L}^T \cdot \mathbf{a}_L^L$ ◎ | $\mathbf{a}_L^L = \left(\mathbf{D}_B^L\right)^I \cdot \mathbf{a}_L^B$ × | $\mathbf{a}_L^B = \left(\mathbf{D}_B^L\right)^{I^T} \cdot \mathbf{a}_L^L$ × |
| 5 | $\mathbf{a}_I^L = \mathbf{D}_A^L \cdot \mathbf{a}_I^A$ ◎ | $\mathbf{a}_I^A = {\mathbf{D}_A^L}^T \cdot \mathbf{a}_I^L$ ◎ | $\mathbf{a}_I^L = \left(\mathbf{D}_A^L\right)^I \cdot \mathbf{a}_I^A$ × | $\mathbf{a}_I^A = \left(\mathbf{D}_A^L\right)^{I^T} \cdot \mathbf{a}_I^L$ × |
| 6 | $\mathbf{a}_L^L = \mathbf{D}_A^L \cdot \mathbf{a}_L^A$ ◎ | $\mathbf{a}_L^A = {\mathbf{D}_A^L}^T \cdot \mathbf{a}_L^L$ ◎ | $\mathbf{a}_L^L = \left(\mathbf{D}_A^L\right)^I \cdot \mathbf{a}_L^A$ × | $\mathbf{a}_L^A = \left(\mathbf{D}_A^L\right)^{I^T} \cdot \mathbf{a}_L^L$ × |
| 7 | $\mathbf{a}_I^I = \mathbf{D}_B^I \cdot \mathbf{a}_I^B$ ◎ | $\mathbf{a}_I^B = {\mathbf{D}_B^I}^T \cdot \mathbf{a}_I^I$ ◎ | $\mathbf{a}_I^I = \left(\mathbf{D}_B^I\right)^I \cdot \mathbf{a}_I^B$ ◎ | $\mathbf{a}_I^B = \left(\mathbf{D}_B^I\right)^{I^T} \cdot \mathbf{a}_I^I$ ◎ |
| 8 | $\mathbf{a}_L^I = \mathbf{D}_B^I \cdot \mathbf{a}_L^B$ ◎ | $\mathbf{a}_L^B = {\mathbf{D}_B^I}^T \cdot \mathbf{a}_L^I$ ◎ | $\mathbf{a}_L^I = \left(\mathbf{D}_B^I\right)^I \cdot \mathbf{a}_L^B$ ◎ | $\mathbf{a}_L^B = \left(\mathbf{D}_B^I\right)^{I^T} \cdot \mathbf{a}_L^I$ ◎ |
| 9 | $\mathbf{a}_I^I = \mathbf{D}_A^I \cdot \mathbf{a}_I^A$ ◎ | $\mathbf{a}_I^A = {\mathbf{D}_A^I}^T \cdot \mathbf{a}_I^I$ ◎ | $\mathbf{a}_I^I = \left(\mathbf{D}_A^I\right)^I \cdot \mathbf{a}_I^A$ ◎ | $\mathbf{a}_I^A = \left(\mathbf{D}_A^I\right)^{I^T} \cdot \mathbf{a}_I^I$ ◎ |
| 10 | $\mathbf{a}_L^I = \mathbf{D}_A^I \cdot \mathbf{a}_L^A$ ◎ | $\mathbf{a}_L^A = {\mathbf{D}_A^I}^T \cdot \mathbf{a}_L^I$ ◎ | $\mathbf{a}_L^I = \left(\mathbf{D}_A^I\right)^I \cdot \mathbf{a}_L^A$ ◎ | $\mathbf{a}_L^A = \left(\mathbf{D}_A^I\right)^{I^T} \cdot \mathbf{a}_L^I$ ◎ |
| 11 | $\mathbf{a}_I^B = \mathbf{D}_A^B \cdot \mathbf{a}_I^A$ ◎ | $\mathbf{a}_I^A = {\mathbf{D}_A^B}^T \cdot \mathbf{a}_I^B$ ◎ | $\mathbf{a}_I^B = \left(\mathbf{D}_A^B\right)^I \cdot \mathbf{a}_I^A$ × | $\mathbf{a}_I^A = \left(\mathbf{D}_A^B\right)^{I^T} \cdot \mathbf{a}_I^B$ × |
| 12 | $\mathbf{a}_L^B = \mathbf{D}_A^B \cdot \mathbf{a}_L^A$ ◎ | $\mathbf{a}_L^A = {\mathbf{D}_A^B}^T \cdot \mathbf{a}_L^B$ ◎ | $\mathbf{a}_L^B = \left(\mathbf{D}_A^B\right)^I \cdot \mathbf{a}_L^A$ × | $\mathbf{a}_L^A = \left(\mathbf{D}_A^B\right)^{I^T} \cdot \mathbf{a}_L^B$ × |
| 13 | | | $\mathbf{a}_L^B = \left(\mathbf{D}_A^B\right)^L \cdot \mathbf{a}_L^A$ × | $\mathbf{a}_L^A = \left(\mathbf{D}_A^B\right)^{L^T} \cdot \mathbf{a}_L^B$ × |

| No. | オイラー角型四元数による座標変換 | | 固定角型四元数による座標変換 | |
|---|---|---|---|---|
| 21 | $\mathbf{a}_B^I = \mathbf{Q}_L^I \cdot \mathbf{a}_B^L \cdot {\mathbf{Q}_L^I}^*$ ◎ | $\mathbf{a}_B^L = {\mathbf{Q}_L^I}^* \cdot \mathbf{a}_B^I \cdot \mathbf{Q}_L^I$ ◎ | $\mathbf{a}_B^I = \left(\mathbf{Q}_L^I\right)^I \cdot \mathbf{a}_B^L \cdot \left(\mathbf{Q}_L^I\right)^{I^*}$ ◎ | $\mathbf{a}_B^L = \left(\mathbf{Q}_L^I\right)^{I^*} \cdot \mathbf{a}_B^I \cdot \left(\mathbf{Q}_L^I\right)^I$ ◎ |
| 22 | $\mathbf{a}_A^I = \mathbf{Q}_L^I \cdot \mathbf{a}_A^L \cdot {\mathbf{Q}_L^I}^*$ ◎ | $\mathbf{a}_A^L = {\mathbf{Q}_L^I}^* \cdot \mathbf{a}_A^I \cdot \mathbf{Q}_L^I$ ◎ | $\mathbf{a}_A^I = \left(\mathbf{Q}_L^I\right)^I \cdot \mathbf{a}_A^L \cdot \left(\mathbf{Q}_L^I\right)^{I^*}$ ◎ | $\mathbf{a}_A^L = \left(\mathbf{Q}_L^I\right)^{I^*} \cdot \mathbf{a}_A^I \cdot \left(\mathbf{Q}_L^I\right)^I$ ◎ |
| 23 | $\mathbf{a}_I^L = \mathbf{Q}_B^L \cdot \mathbf{a}_I^B \cdot {\mathbf{Q}_B^L}^*$ ◎ | $\mathbf{a}_I^B = {\mathbf{Q}_B^L}^* \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_B^L$ ◎ | $\mathbf{a}_I^L = \left(\mathbf{Q}_B^L\right)^I \cdot \mathbf{a}_I^B \cdot \left(\mathbf{Q}_B^L\right)^{I^*}$ × | $\mathbf{a}_I^B = \left(\mathbf{Q}_B^L\right)^{I^*} \cdot \mathbf{a}_I^L \cdot \left(\mathbf{Q}_B^L\right)^I$ × |
| 24 | $\mathbf{a}_L^L = \mathbf{Q}_B^L \cdot \mathbf{a}_L^B \cdot {\mathbf{Q}_B^L}^*$ ◎ | $\mathbf{a}_L^B = {\mathbf{Q}_B^L}^* \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_B^L$ ◎ | $\mathbf{a}_L^L = \left(\mathbf{Q}_B^L\right)^I \cdot \mathbf{a}_L^B \cdot \left(\mathbf{Q}_B^L\right)^{I^*}$ × | $\mathbf{a}_L^B = \left(\mathbf{Q}_B^L\right)^{I^*} \cdot \mathbf{a}_L^L \cdot \left(\mathbf{Q}_B^L\right)^I$ × |
| 25 | $\mathbf{a}_I^L = \mathbf{Q}_A^L \cdot \mathbf{a}_I^A \cdot {\mathbf{Q}_A^L}^*$ ◎ | $\mathbf{a}_I^A = {\mathbf{Q}_A^L}^* \cdot \mathbf{a}_I^L \cdot \mathbf{Q}_A^L$ ◎ | $\mathbf{a}_I^L = \left(\mathbf{Q}_A^L\right)^I \cdot \mathbf{a}_I^A \cdot \left(\mathbf{Q}_A^L\right)^{I^*}$ × | $\mathbf{a}_I^A = \left(\mathbf{Q}_A^L\right)^{I^*} \cdot \mathbf{a}_I^L \cdot \left(\mathbf{Q}_A^L\right)^I$ × |
| 26 | $\mathbf{a}_L^L = \mathbf{Q}_A^L \cdot \mathbf{a}_L^A \cdot {\mathbf{Q}_A^L}^*$ ◎ | $\mathbf{a}_L^A = {\mathbf{Q}_A^L}^* \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_A^L$ ◎ | $\mathbf{a}_L^L = \left(\mathbf{Q}_A^L\right)^I \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^L\right)^{I^*}$ × | $\mathbf{a}_L^A = \left(\mathbf{Q}_A^L\right)^{I^*} \cdot \mathbf{a}_L^L \cdot \left(\mathbf{Q}_A^L\right)^I$ × |
| 27 | $\mathbf{a}_I^I = \mathbf{Q}_B^I \cdot \mathbf{a}_I^B \cdot {\mathbf{Q}_B^I}^*$ ◎ | $\mathbf{a}_I^B = {\mathbf{Q}_B^I}^* \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_B^I$ ◎ | $\mathbf{a}_I^I = \left(\mathbf{Q}_B^I\right)^I \cdot \mathbf{a}_I^B \cdot \left(\mathbf{Q}_B^I\right)^{I^*}$ ◎ | $\mathbf{a}_I^B = \left(\mathbf{Q}_B^I\right)^{I^*} \cdot \mathbf{a}_I^I \cdot \left(\mathbf{Q}_B^I\right)^I$ ◎ |
| 28 | $\mathbf{a}_L^I = \mathbf{Q}_B^I \cdot \mathbf{a}_L^B \cdot {\mathbf{Q}_B^I}^*$ ◎ | $\mathbf{a}_L^B = {\mathbf{Q}_B^I}^* \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_B^I$ ◎ | $\mathbf{a}_L^I = \left(\mathbf{Q}_B^I\right)^I \cdot \mathbf{a}_L^B \cdot \left(\mathbf{Q}_B^I\right)^{I^*}$ ◎ | $\mathbf{a}_L^B = \left(\mathbf{Q}_B^I\right)^{I^*} \cdot \mathbf{a}_L^I \cdot \left(\mathbf{Q}_B^I\right)^I$ ◎ |
| 29 | $\mathbf{a}_I^I = \mathbf{Q}_A^I \cdot \mathbf{a}_I^A \cdot {\mathbf{Q}_A^I}^*$ ◎ | $\mathbf{a}_I^A = {\mathbf{Q}_A^I}^* \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_A^I$ ◎ | $\mathbf{a}_I^I = \left(\mathbf{Q}_A^I\right)^I \cdot \mathbf{a}_I^A \cdot \left(\mathbf{Q}_A^I\right)^{I^*}$ ◎ | $\mathbf{a}_I^A = \left(\mathbf{Q}_A^I\right)^{I^*} \cdot \mathbf{a}_I^I \cdot \left(\mathbf{Q}_A^I\right)^I$ ◎ |
| 30 | $\mathbf{a}_L^I = \mathbf{Q}_A^I \cdot \mathbf{a}_L^A \cdot {\mathbf{Q}_A^I}^*$ ◎ | $\mathbf{a}_L^A = {\mathbf{Q}_A^I}^* \cdot \mathbf{a}_L^I \cdot \mathbf{Q}_A^I$ ◎ | $\mathbf{a}_L^I = \left(\mathbf{Q}_A^I\right)^I \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^I\right)^{I^*}$ ◎ | $\mathbf{a}_L^A = \left(\mathbf{Q}_A^I\right)^{I^*} \cdot \mathbf{a}_L^I \cdot \left(\mathbf{Q}_A^I\right)^I$ ◎ |
| 31 | $\mathbf{a}_I^B = \mathbf{Q}_A^B \cdot \mathbf{a}_I^A \cdot {\mathbf{Q}_A^B}^*$ ◎ | $\mathbf{a}_I^A = {\mathbf{Q}_A^B}^* \cdot \mathbf{a}_I^B \cdot \mathbf{Q}_A^B$ ◎ | $\mathbf{a}_I^B = \left(\mathbf{Q}_A^B\right)^I \cdot \mathbf{a}_I^A \cdot \left(\mathbf{Q}_A^B\right)^{I^*}$ × | $\mathbf{a}_I^A = \left(\mathbf{Q}_A^B\right)^{I^*} \cdot \mathbf{a}_I^B \cdot \left(\mathbf{Q}_A^B\right)^I$ × |
| 32 | $\mathbf{a}_L^B = \mathbf{Q}_A^B \cdot \mathbf{a}_L^A \cdot {\mathbf{Q}_A^B}^*$ ◎ | $\mathbf{a}_L^A = {\mathbf{Q}_A^B}^* \cdot \mathbf{a}_L^B \cdot \mathbf{Q}_A^B$ ◎ | $\mathbf{a}_L^B = \left(\mathbf{Q}_A^B\right)^I \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^B\right)^{I^*}$ × | $\mathbf{a}_L^A = \left(\mathbf{Q}_A^B\right)^{I^*} \cdot \mathbf{a}_L^B \cdot \left(\mathbf{Q}_A^B\right)^I$ × |
| 33 | | | $\mathbf{a}_L^B = \left(\mathbf{Q}_A^B\right)^L \cdot \mathbf{a}_L^A \cdot \left(\mathbf{Q}_A^B\right)^{L^*}$ × | $\mathbf{a}_L^A = \left(\mathbf{Q}_A^B\right)^{L^*} \cdot \mathbf{a}_L^B \cdot \left(\mathbf{Q}_A^B\right)^L$ × |

ベクトルの座標変換は方向余弦マトリクスまたはオイラー角型四元数を用いて、基準座標系の如何に依らず変換できる。

方向余弦マトリクスによる座標系の変換は、右下添え字側の座標系から右上添え字側の座標系へ切り替わる。座標系Gから見たベクトルを座標系Aから見たベクトルに座標変換するのは次のようになる。

$\mathbf{a}^A = \mathbf{D}_G^A \cdot \mathbf{a}^G$ :ベクトルの座標変換

(1) 方向余弦マトリクスによる座標変換

ベクトルの座標変換の式は次のように証明される。

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{A}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{A}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$

(2) 固定角型回転マトリクスによる座標変換

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}$$
$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}^{\mathrm{T}}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}}\right)^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}}$$

(3) オイラー角型四元数による座標変換

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^{*}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^{*}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^{*}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^{*}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}^{*}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}^{*}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}\,*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$

(4) 固定角型四元数による座標変換

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}$$

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{B}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{a}_{\mathrm{A}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^{*}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}\right)^{*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^{*}} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^{*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{A}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{A}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}}$$

$$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{B}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}\right)^* \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}}$$

$$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*}\right)^* \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}^*} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}^*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{B}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}^{\mathrm{T}}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}}$$

# 6 まとめ

## 6.1 座標系の回転

方向余弦マトリクスは基準座標系の如何に関わらず利用できる。他の体（非可換体）においては、それらを表現する基準の座標系と演算対象の座標軸ベクトルを表現する基準の座標系が一致していれば利用できる。

$\mathbf{x}_B^I = \mathbf{x}_L^I \cdot \mathbf{D}_B^L \Rightarrow \left(\underline{x}_B^I \quad \underline{y}_B^I \quad \underline{z}_B^I\right) = \left(\underline{x}_L^I \quad \underline{y}_L^I \quad \underline{z}_L^I\right) \cdot \mathbf{D}_B^L$ ：方向余弦マトリクスによる座標系の回転

$$\left.\begin{aligned} \underline{x}_B^I &= \mathbf{Q}_B^I \cdot \underline{x} \cdot \mathbf{Q}_B^{I\,*} \\ \underline{y}_B^I &= \mathbf{Q}_B^I \cdot \underline{y} \cdot \mathbf{Q}_B^{I\,*} \\ \underline{z}_B^I &= \mathbf{Q}_B^I \cdot \underline{z} \cdot \mathbf{Q}_B^{I\,*} \end{aligned}\right\}$$ ：オイラー角型四元数による座標系の回転

$\mathbf{x}_B^I = \left(\mathbf{D}_B^L\right)^I \cdot \mathbf{x}_L^I \Rightarrow \left(\underline{x}_B^I \quad \underline{y}_B^I \quad \underline{z}_B^I\right) = \left(\mathbf{D}_B^L\right)^I \cdot \left(\underline{x}_L^I \quad \underline{y}_L^I \quad \underline{z}_L^I\right)$ ：固定角型回転マトリクスよる座標系の回転

$$\left.\begin{aligned} \underline{x}_B^I &= \left(\mathbf{Q}_B^I\right)^I \cdot \underline{x} \cdot \left(\mathbf{Q}_B^I\right)^{I*} \\ \underline{y}_B^I &= \left(\mathbf{Q}_B^I\right)^I \cdot \underline{y} \cdot \left(\mathbf{Q}_B^I\right)^{I*} \\ \underline{z}_B^I &= \left(\mathbf{Q}_B^I\right)^I \cdot \underline{z} \cdot \left(\mathbf{Q}_B^I\right)^{I*} \end{aligned}\right\}$$ ：固定角型四元数による座標系の回転

## 6.2 ベクトルの回転

4種の体（非可換体）はいずれも基準座標系が一致していれば利用できる。

$\mathbf{a}_B^I = \mathbf{D}_B^I \cdot \mathbf{a}_I^I$ ：方向余弦マトリクスによるベクトルの回転

$\mathbf{a}_B^I = \mathbf{Q}_B^I \cdot \mathbf{a}_I^I \cdot \mathbf{Q}_B^{I\,*}$ ：オイラー角型四元数によるベクトルの回転

$\mathbf{a}_B^I = \left(\mathbf{D}_A^B\right)^{I^T} \cdot \mathbf{a}_A^I$ ：固定角型回転マトリクスによるベクトルの回転

$\mathbf{a}_B^I = \left(\mathbf{Q}_A^B\right)^{I^*} \cdot \mathbf{a}_A^I \cdot \left(\mathbf{Q}_A^B\right)^I$ ：固定角型四元数によるベクトルの回転

## 6.3 ベクトルの座標変換

方向余弦マトリクスおよびオイラー角型四元数は、基準座標系の如何に関わらず利用できる。

$$\mathbf{a}_B^I = \mathbf{D}_L^I \cdot \mathbf{a}_B^L \Rightarrow \begin{pmatrix} a_{x_B}^I \\ a_{y_B}^I \\ a_{z_B}^I \end{pmatrix} = \mathbf{D}_L^I \cdot \begin{pmatrix} a_{x_B}^L \\ a_{y_B}^L \\ a_{z_B}^L \end{pmatrix}$$ ：方向余弦マトリクスによるベクトルの座標変換

$$\mathbf{a}_B^I = \mathbf{Q}_L^I \otimes \mathbf{a}_B^L \otimes \mathbf{Q}_L^{I\,*} \Rightarrow \begin{pmatrix} a_{x_B}^I \\ a_{y_B}^I \\ a_{z_B}^I \end{pmatrix} = \mathbf{Q}_L^I \otimes \begin{pmatrix} 0 \\ a_{x_B}^L \\ a_{y_B}^L \\ a_{z_B}^L \end{pmatrix} \otimes \mathbf{Q}_L^{I\,*}$$ ：オイラー角型四元数によるベクトルの座標変換

固定角型回転マトリクスおよび固定角型四元数は、それらをオイラー角型の方向余弦マトリクスおよび四元数に変形できる場合、座標変換に利用できる。ただし、この体（非可換体）は座標変換に不向きであり、わざわざ利用するメリットは無い。

$$\mathbf{a}_B^I = \left(\mathbf{D}_L^I\right)^I \cdot \mathbf{a}_B^L = \left(\mathbf{D}_L^I \cdot \mathbf{D}_I^{I^T}\right) \cdot \mathbf{a}_B^L = \mathbf{D}_L^I \cdot \mathbf{a}_B^L = \mathbf{a}_B^I$$

$$\Rightarrow \begin{pmatrix} a_{x_B}^I \\ a_{y_B}^I \\ a_{z_B}^I \end{pmatrix} = \left(\mathbf{D}_L^I\right)^I \cdot \begin{pmatrix} a_{x_B}^L \\ a_{y_B}^L \\ a_{z_B}^L \end{pmatrix}$$ ：固定角型回転マトリクスによるベクトルの座標変換

$$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \otimes \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \otimes \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right) \otimes \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \otimes \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right)^* = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^*}\right) \otimes \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \otimes \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*}\right)$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \otimes \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$$
$$\Rightarrow \begin{pmatrix} a_{x_B}^{I} \\ a_{y_B}^{I} \\ a_{z_B}^{I} \end{pmatrix} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \otimes \begin{pmatrix} 0 \\ a_{x_B}^{L} \\ a_{y_B}^{L} \\ a_{z_B}^{L} \end{pmatrix} \otimes \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} : \text{固定角型四元数によるベクトルの座標変換}$$

## 7 固定角型回転マトリクスおよび固定角型四元数の基準座標系の変換

固定角型回転マトリクスの基準座標系は、「慣性航法のアルゴリズム」に示した式(1.2-13)のように、他の座標系に変換できる。下記に例を示す。

$$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$
$$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

固定角型四元数も固定角型回転マトリクスと同様に、次のように変換できる。

$$\left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \otimes \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} = \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\} \otimes \frac{1}{2}\left\{-1 + \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}\right\} \otimes \frac{1}{\sqrt{2}}\left\{1 - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$
$$= \frac{1}{4}\left\{-1 + \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - 1\right\} \otimes \left\{1 - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\} = \frac{1}{4}\left\{-2 + \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}\right\} \otimes \left\{1 - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$
$$= \frac{1}{4}\left\{-2 + \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}\right\} \otimes \left\{1 - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\} = \frac{1}{4}\left\{-2 + \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}\right\} = \frac{1}{2}\left\{-1 + \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right\}$$
$$\left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{L}} \otimes \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{I}}^{\mathrm{L}^*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}^*} \otimes \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \otimes \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} = \frac{1}{\sqrt{2}}\left\{1 - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\} \otimes \frac{1}{2}\left\{-1 + \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right\} \otimes \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$
$$= \frac{1}{4}\left\{-1 + \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} + 1\right\} \otimes \left\{1 + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\} = \frac{1}{4}\left\{0 + \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}\right\} \otimes \left\{1 + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$
$$= \frac{1}{4}\left\{0 + \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}\right\} \otimes \left\{1 + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\} = \frac{1}{4}\left\{0 + \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} - 2\right\} = \frac{1}{2}\left\{-1 + \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}\right\}$$