座標系の回転とベクトルの回転および座標変換の例題

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下記の4種類の体（非可換体）について演算結果を比較検討する。

(1) $\mathbf{D}_{\mathrm{R}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{R}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{R}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{R}}^{\mathrm{I}}\right) = [\theta_1]_{\mathrm{i}} \cdot [\theta_2]_{\mathrm{j}} \cdot [\theta_3]_{\mathrm{k}}$ ：方向余弦マトリクス

(2) $\left(\mathbf{D}_{\mathrm{G}}^{\mathrm{R}}\right)^{\mathrm{I}} = \mathbf{D}_{\mathrm{G}}^{\mathrm{I}} \cdot {\mathbf{D}_{\mathrm{R}}^{\mathrm{I}}}^{\mathrm{T}} = \left(\underline{\mathrm{x}}_{\mathrm{G}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{G}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{G}}^{\mathrm{I}}\right) \cdot \left(\underline{\mathrm{x}}_{\mathrm{R}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{R}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{R}}^{\mathrm{I}}\right)^{\mathrm{T}}$ ：固定角型回転マトリクス

(3) $\mathbf{Q}_{\mathrm{R}}^{\mathrm{I}} = \mathbf{Q}(\theta_1, \underline{\mathrm{e}}_{\mathrm{i}}) \otimes \mathbf{Q}(\theta_2, \underline{\mathrm{e}}_{\mathrm{j}}) \otimes \mathbf{Q}(\theta_3, \underline{\mathrm{e}}_{\mathrm{k}})$ ：オイラー角型四元数

(4) $\left(\mathbf{Q}_{\mathrm{G}}^{\mathrm{R}}\right)^{\mathrm{I}} = \mathbf{Q}_{\mathrm{G}}^{\mathrm{I}} \otimes {\mathbf{Q}_{\mathrm{R}}^{\mathrm{I}}}^{*}$ ：固定角型四元数

演算内容は下記の3種類。

(1) 座標系の回転
(2) ベクトルの回転
(3) ベクトルの座標変換

1 座標系

基準とする慣性座標系I（o-x，y，z）の軸単位ベクトルを $(\underline{x}\quad \underline{y}\quad \underline{z})$ とし、慣性座標系Iを回転してできた局地座標系L（o-x_L，y_L，z_L）の軸単位ベクトルを慣性座標系Iで表して $(\underline{x}_L^I\quad \underline{y}_L^I\quad \underline{z}_L^I)$ とする。さらに、局地座標系Lを基準として2つの座標系を作り、それぞれ座標系B（o-x_B，y_B，z_B）および座標系A（o-x_A，y_A，z_A）とし、それぞれの軸単位ベクトルを座標系Lで表して $(\underline{x}_B^L\quad \underline{y}_B^L\quad \underline{z}_B^L)$および $(\underline{x}_A^L\quad \underline{y}_A^L\quad \underline{z}_A^L)$ とする。また、これらは慣性座標系Iでは$(\underline{x}_B^I\quad \underline{y}_B^I\quad \underline{z}_B^I)$および $(\underline{x}_A^I\quad \underline{y}_A^I\quad \underline{z}_A^I)$ と表す。

慣性座標系Iは地球中心に原点を置き、グリニッジ子午線と赤道面の交点方向に x 軸、自転軸方向に z 軸、それらと右手系を成す方向に y 軸を定義する。

局地座標系Lは慣性座標系Iの第3軸周りに東経 λ だけ回転し、新しくできた第2軸周りに緯度 ϕ の負符号の回転、さらに新しくできた第1軸周りに北から時計回りに測った方位角 μ の回転を施した座標系とする。これにより x_L 軸は局所鉛直方向、y_L 軸と z_L 軸は局所水平面にあり、z_L 軸が方位角方向を向く。

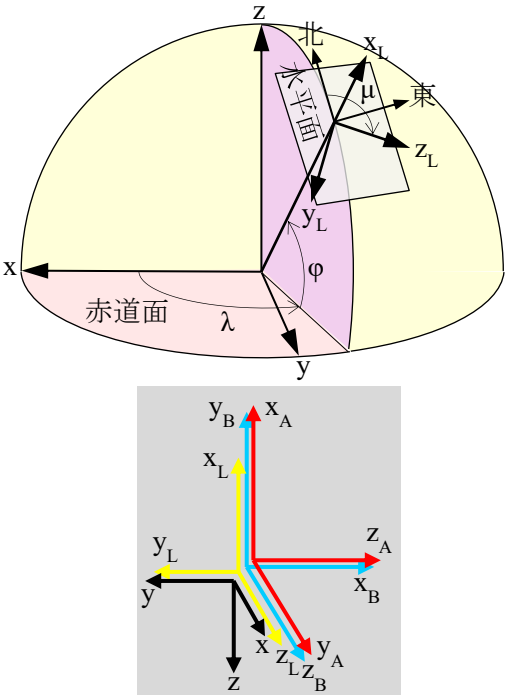


図1-1　例題の座標系

$(\underline{x}_L^I\quad \underline{y}_L^I\quad \underline{z}_L^I) = (\underline{x}\quad \underline{y}\quad \underline{z})\cdot[\lambda]_3\cdot[-\varphi]_2\cdot[-\mu]_1 = (\underline{x}\quad \underline{y}\quad \underline{z})\cdot \mathbf{D}_L^I$ ：座標系L

ここで、$\lambda = 90°,\ \varphi = -90°,\ \mu = -90°$ とする。また、$(\underline{x}\quad \underline{y}\quad \underline{z})$は単位マトリクスである。

$$\mathbf{D}_L^I = [90°]_3\cdot[90°]_2\cdot[90°]_1 = \begin{pmatrix}0&-1&0\\1&0&0\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}0&0&1\\0&1&0\\-1&0&0\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&0&-1\\0&1&0\end{pmatrix}$$

$$= \begin{pmatrix}0&0&1\\0&1&0\\-1&0&0\end{pmatrix}$$ ：座標系Iから見た座標系Lの方向余弦マトリクス

$$\underline{x}_L^I = \begin{pmatrix}0\\0\\-1\end{pmatrix},\ \underline{y}_L^I = \begin{pmatrix}0\\1\\0\end{pmatrix},\ \underline{z}_L^I = \begin{pmatrix}1\\0\\0\end{pmatrix}$$ ：座標系Iから見た座標系Lの軸ベクトル

座標系Iから座標系Lへの回転を表す四元数はオイラー角による回転順に合成して、次のように表される。

$$\mathbf{Q}_{L1} = \cos\frac{\lambda}{2} + \sin\frac{\lambda}{2}\underline{e}_3 = \cos(45°) + \sin(45°)\begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$$

$$\mathbf{Q}_{L2} = \cos\frac{-\varphi}{2} + \sin\frac{-\varphi}{2}\underline{e}_2 = \cos(45°) + \sin(45°)\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$$

$$\mathbf{Q}_{L3} = \cos\frac{-\mu}{2} + \sin\frac{-\mu}{2}\underline{e}_1 = \cos(45°) + \sin(45°)\begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix}1\\0\\0\end{pmatrix}\right\}$$

$$\mathbf{Q}_L^I = \mathbf{Q}_{L1}\otimes\mathbf{Q}_{L2}\otimes\mathbf{Q}_{L3} = \frac{1}{2\sqrt{2}}\left\{1 + \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}\otimes\left\{1 + \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}\otimes\left\{1 + \begin{pmatrix}1\\0\\0\end{pmatrix}\right\}$$

$$= \frac{1}{2\sqrt{2}}\left\{1 + \begin{pmatrix}0\\0\\1\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix} + \begin{pmatrix}-1\\0\\0\end{pmatrix}\right\} \otimes \left\{1 + \begin{pmatrix}1\\0\\0\end{pmatrix}\right\} = \frac{1}{2\sqrt{2}}\left\{1 + \begin{pmatrix}-1\\1\\1\end{pmatrix}\right\} \otimes \left\{1 + \begin{pmatrix}1\\0\\0\end{pmatrix}\right\}$$

$$= \frac{1}{2\sqrt{2}}\left\{1 + \begin{pmatrix}-1\\1\\1\end{pmatrix} + \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\-1\end{pmatrix} + 1\right\} = \frac{1}{2\sqrt{2}}\left\{2 + \begin{pmatrix}0\\2\\0\end{pmatrix}\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$$:座標系Iから座標系Lへの回転を表す四元数

座標系Bは座標系Lの第2軸周りにθ_1回転し、新しくできた第3軸周りにθ_2回転し、さらに新しくできた第1軸周りにθ_3回転した座標系とする。

$\left(\underline{x}_B^L \quad \underline{y}_B^L \quad \underline{z}_B^L\right) = \left(\underline{x}_L^L \quad \underline{y}_L^L \quad \underline{z}_L^L\right) \cdot [\theta_1]_2 \cdot [\theta_2]_3 \cdot [\theta_3]_1 = \left(\underline{x}_L^L \quad \underline{y}_L^L \quad \underline{z}_L^L\right) \cdot \mathbf{D}_B^L$:座標系B

ここで、$\theta_1 = -90°,\ \theta_2 = -90°,\ \theta_3 = -90°$ とする。また、$\left(\underline{x}_L^L \quad \underline{y}_L^L \quad \underline{z}_L^L\right)$ は単位マトリクスである。

$$\mathbf{D}_B^L = [-90°]_2 \cdot [-90°]_3 \cdot [-90°]_1 = \begin{pmatrix}0 & 0 & -1\\0 & 1 & 0\\1 & 0 & 0\end{pmatrix} \cdot \begin{pmatrix}0 & 1 & 0\\-1 & 0 & 0\\0 & 0 & 1\end{pmatrix} \cdot \begin{pmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{pmatrix}$$

$$= \begin{pmatrix}0 & 1 & 0\\-1 & 0 & 0\\0 & 0 & 1\end{pmatrix}$$:座標系Lから見た座標系Bの方向余弦マトリクス

$$\mathbf{D}_B^I = \mathbf{D}_L^I \cdot \mathbf{D}_B^L = \begin{pmatrix}0 & 0 & 1\\0 & 1 & 0\\-1 & 0 & 0\end{pmatrix} \cdot \begin{pmatrix}0 & 1 & 0\\-1 & 0 & 0\\0 & 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}0 & 0 & 1\\-1 & 0 & 0\\0 & -1 & 0\end{pmatrix}$$:座標系Iから見た座標系Bの方向余弦マトリクス

$$\underline{x}_B^L = \begin{pmatrix}0\\-1\\0\end{pmatrix},\ \underline{y}_B^L = \begin{pmatrix}1\\0\\0\end{pmatrix},\ \underline{z}_B^L = \begin{pmatrix}0\\0\\1\end{pmatrix}$$:座標系Lから見た座標系Bの軸ベクトル

$$\underline{x}_B^I = \begin{pmatrix}0\\-1\\0\end{pmatrix},\ \underline{y}_B^I = \begin{pmatrix}0\\0\\-1\end{pmatrix},\ \underline{z}_B^I = \begin{pmatrix}1\\0\\0\end{pmatrix}$$:座標系Iから見た座標系Bの軸ベクトル

座標系Lから座標系Bへの回転を表す四元数はオイラー角による回転順に合成して、次のように表される。

$$\mathbf{Q}_{B1} = \cos\frac{\theta_1}{2} + \sin\frac{\theta_1}{2}\underline{e}_2 = \cos(-45°) + \sin(-45°)\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix}0\\-1\\0\end{pmatrix}\right\}$$

$$\mathbf{Q}_{B2} = \cos\frac{\theta_2}{2} + \sin\frac{\theta_2}{2}\underline{e}_3 = \cos(-45°) + \sin(-45°)\begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix}0\\0\\-1\end{pmatrix}\right\}$$

$$\mathbf{Q}_{B3} = \cos\frac{\theta_3}{2} + \sin\frac{\theta_3}{2}\underline{e}_1 = \cos(-45°) + \sin(-45°)\begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}$$

$$\mathbf{Q}_B^L = \mathbf{Q}_{B1} \otimes \mathbf{Q}_{B2} \otimes \mathbf{Q}_{B3} = \frac{1}{2\sqrt{2}}\left\{1 + \begin{pmatrix}0\\-1\\0\end{pmatrix}\right\} \otimes \left\{1 + \begin{pmatrix}0\\0\\-1\end{pmatrix}\right\} \otimes \left\{1 + \begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}$$

$$= \frac{1}{2\sqrt{2}}\left\{1 + \begin{pmatrix}0\\-1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-1\end{pmatrix} + \begin{pmatrix}1\\0\\0\end{pmatrix}\right\} \otimes \left\{1 + \begin{pmatrix}-1\\0\\0\end{pmatrix}\right\} = \frac{1}{2\sqrt{2}}\left\{1 + \begin{pmatrix}1\\-1\\-1\end{pmatrix}\right\} \otimes \left\{1 + \begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}$$

$$= \frac{1}{2\sqrt{2}}\left\{1 + \begin{pmatrix}1\\-1\\-1\end{pmatrix} + \begin{pmatrix}-1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\-1\end{pmatrix} + 1\right\} = \frac{1}{2\sqrt{2}}\left\{2 + \begin{pmatrix}0\\0\\-2\end{pmatrix}\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix}0\\0\\-1\end{pmatrix}\right\}$$:座標系Lから座標系Bへの回転を表す四元数

$$\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}=\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\otimes\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}=\frac{1}{\sqrt{2}}\left\{1+\begin{pmatrix}0\\1\\0\end{pmatrix}\right\}\otimes\frac{1}{\sqrt{2}}\left\{1+\begin{pmatrix}0\\0\\-1\end{pmatrix}\right\}=\frac{1}{2}\left\{1+\begin{pmatrix}0\\1\\0\end{pmatrix}+\begin{pmatrix}0\\0\\-1\end{pmatrix}+\begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}$$

$$=\frac{1}{2}\left\{1+\begin{pmatrix}-1\\1\\-1\end{pmatrix}\right\}$$:座標系Iから座標系Bへの回転を表す四元数

座標系Aは座標系Lの第3軸周りにφ_1回転し、新しくできた第1軸周りにφ_2回転し、さらに新しくできた第3軸周りにφ_3回転した座標系とする。

$$\left(\underline{\mathrm{x}}_{\mathrm{A}}^{\mathrm{L}}\quad\underline{\mathrm{y}}_{\mathrm{A}}^{\mathrm{L}}\quad\underline{\mathrm{z}}_{\mathrm{A}}^{\mathrm{L}}\right)=\left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{L}}\quad\underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{L}}\quad\underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{L}}\right)\cdot[\varphi_1]_3\cdot[\varphi_2]_1\cdot[\varphi_3]_3=\left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{L}}\quad\underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{L}}\quad\underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{L}}\right)\cdot\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}$$:座標系A

ここで、 $\varphi_1=180°,\ \varphi_2=-90°,\ \varphi_3=180°$ とする。また、$\left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{L}}\quad\underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{L}}\quad\underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{L}}\right)$ は単位マトリクスである。

$$\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}=[180°]_3\cdot[-90°]_1\cdot[180°]_3=\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&0&1\\0&-1&0\end{pmatrix}\cdot\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&1\end{pmatrix}$$

$$=\begin{pmatrix}1&0&0\\0&0&-1\\0&1&0\end{pmatrix}$$:座標系Lから見た座標系Aの方向余弦マトリクス

$$\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}=\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\cdot\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}=\begin{pmatrix}0&0&1\\0&1&0\\-1&0&0\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&0&-1\\0&1&0\end{pmatrix}$$

$$=\begin{pmatrix}0&1&0\\0&0&-1\\-1&0&0\end{pmatrix}$$:座標系Iから見た座標系Aの方向余弦マトリクス

$$\underline{\mathrm{x}}_{\mathrm{A}}^{\mathrm{L}}=\begin{pmatrix}1\\0\\0\end{pmatrix},\ \underline{\mathrm{y}}_{\mathrm{A}}^{\mathrm{L}}=\begin{pmatrix}0\\0\\1\end{pmatrix},\ \underline{\mathrm{z}}_{\mathrm{A}}^{\mathrm{L}}=\begin{pmatrix}0\\-1\\0\end{pmatrix}$$:座標系Lから見た座標系Aの軸ベクトル

$$\underline{\mathrm{x}}_{\mathrm{A}}^{\mathrm{I}}=\begin{pmatrix}0\\0\\-1\end{pmatrix},\ \underline{\mathrm{y}}_{\mathrm{A}}^{\mathrm{I}}=\begin{pmatrix}1\\0\\0\end{pmatrix},\ \underline{\mathrm{z}}_{\mathrm{A}}^{\mathrm{I}}=\begin{pmatrix}0\\-1\\0\end{pmatrix}$$:座標系Iから見た座標系Aの軸ベクトル

$$\mathbf{Q}_{\mathrm{A1}}=\cos\frac{\varphi_1}{2}+\sin\frac{\varphi_1}{2}\underline{\mathrm{e}}_3=\cos(90°)+\sin(90°)\begin{pmatrix}0\\0\\1\end{pmatrix}=0+\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\mathbf{Q}_{\mathrm{A2}}=\cos\frac{\varphi_2}{2}+\sin\frac{\varphi_2}{2}\underline{\mathrm{e}}_1=\cos(-45°)+\sin(-45°)\begin{pmatrix}1\\0\\0\end{pmatrix}=\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\end{pmatrix}=\frac{1}{\sqrt{2}}\left\{1+\begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}$$

$$\mathbf{Q}_{\mathrm{A3}}=\cos\frac{\varphi_3}{2}+\sin\frac{\varphi_3}{2}\underline{\mathrm{e}}_3=\cos(90°)+\sin(90°)\begin{pmatrix}0\\0\\1\end{pmatrix}=0+\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}=\mathbf{Q}_{\mathrm{A1}}\otimes\mathbf{Q}_{\mathrm{A2}}\otimes\mathbf{Q}_{\mathrm{A3}}=\frac{1}{\sqrt{2}}\left\{0+\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}\otimes\left\{1+\begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}\otimes\left\{0+\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$$

$$=\frac{1}{\sqrt{2}}\left\{0+\begin{pmatrix}0\\0\\1\end{pmatrix}+\begin{pmatrix}0\\-1\\0\end{pmatrix}\right\}\otimes\left\{0+\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}=\frac{1}{\sqrt{2}}\left\{0+\begin{pmatrix}0\\-1\\1\end{pmatrix}\right\}\otimes\left\{0+\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}=\frac{1}{\sqrt{2}}\left\{0+\begin{pmatrix}-1\\0\\0\end{pmatrix}-1\right\}$$

$$=\frac{1}{\sqrt{2}}\left\{-1+\begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}$$:座標系Lから座標Aへの回転を表す四元数

$$\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}=\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\otimes\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}=\frac{1}{\sqrt{2}}\left\{1+\begin{pmatrix}0\\1\\0\end{pmatrix}\right\}\otimes\frac{1}{\sqrt{2}}\left\{-1+\begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}=\frac{1}{2}\left\{-1-\begin{pmatrix}0\\1\\0\end{pmatrix}+\begin{pmatrix}-1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$$

$$=\frac{1}{2}\left\{-1+\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right\}$$:座標系Iから座標Aへの回転を表す四元数

次に座標系Bと座標系Aの関係を示す。

$\left(\underline{x}_A^L \quad \underline{y}_A^L \quad \underline{z}_A^L\right) = \left(\underline{x}_B^L \quad \underline{y}_B^L \quad \underline{z}_B^L\right) \cdot \mathbf{D}_A^B$:オイラー角による座標系の回転

$$\mathbf{D}_A^B = \mathbf{D}_B^{L\,T} \cdot \mathbf{D}_A^L$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}^T \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$:座標系Bから見た座標系Aの方向余弦マトリクス

$$\mathbf{Q}_A^B = \mathbf{Q}_B^{L\,*} \otimes \mathbf{Q}_A^L = \frac{1}{\sqrt{2}}\left\{1 - \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\right\} \otimes \frac{1}{\sqrt{2}}\left\{-1 + \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}\right\} = \frac{1}{2}\left\{-1 + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$

$$= \frac{1}{2}\left\{-1 - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\}$$:座標系Bから座標系Aへの回転を表す四元数

この四元数を方向余弦マトリクスに変換してみると下記のようになる。

$$\mathbf{D}_A^B = \begin{pmatrix} \frac{1}{4}+\frac{1}{4}(1-1-1) & \frac{1}{2}-\frac{1}{2} & \frac{1}{2}+\frac{1}{2} \\ \frac{1}{2}+\frac{1}{2} & \frac{1}{4}+\frac{1}{4}(1-1-1) & \frac{1}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{1}{2} & \frac{1}{2}+\frac{1}{2} & \frac{1}{4}+\frac{1}{4}(1-1-1) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

固定角型回転マトリクスは基準座標系が等しい2つの方向余弦マトリクスを合成したものだから、その基準座標系が固定角型回転マトリクスの基準座標系となり、カッコの外に右上添え字として基準座標系を表示して次のように定義する。

$$\left(\mathbf{D}_L^I\right)^I = \left(\underline{x}_L^I \quad \underline{y}_L^I \quad \underline{z}_L^I\right) \cdot \left(\underline{x}_I^I \quad \underline{y}_I^I \quad \underline{z}_I^I\right)^T = \mathbf{D}_L^I \cdot \mathbf{D}_I^{I\,T} = \mathbf{D}_L^I = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\left(\mathbf{D}_B^L\right)^I = \left(\underline{x}_B^I \quad \underline{y}_B^I \quad \underline{z}_B^I\right) \cdot \left(\underline{x}_L^I \quad \underline{y}_L^I \quad \underline{z}_L^I\right)^T = \mathbf{D}_B^I \cdot \mathbf{D}_L^{I\,T} = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\left(\mathbf{D}_A^L\right)^I = \left(\underline{x}_A^I \quad \underline{y}_A^I \quad \underline{z}_A^I\right) \cdot \left(\underline{x}_L^I \quad \underline{y}_L^I \quad \underline{z}_L^I\right)^T = \mathbf{D}_A^I \cdot \mathbf{D}_L^{I\,T} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\mathbf{D}_B^I\right)^I = \left(\underline{x}_B^I \quad \underline{y}_B^I \quad \underline{z}_B^I\right) \cdot \left(\underline{x}_I^I \quad \underline{y}_I^I \quad \underline{z}_I^I\right)^T = \mathbf{D}_B^I \cdot \mathbf{D}_I^{I\,T} = \mathbf{D}_B^I = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\left(\mathbf{D}_A^I\right)^I = \left(\underline{x}_A^I \quad \underline{y}_A^I \quad \underline{z}_A^I\right) \cdot \left(\underline{x}_I^I \quad \underline{y}_I^I \quad \underline{z}_I^I\right)^T = \mathbf{D}_A^I \cdot \mathbf{D}_I^{I\,T} = \mathbf{D}_A^I = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\left(\mathbf{D}_A^B\right)^I = \left(\underline{x}_A^I \quad \underline{y}_A^I \quad \underline{z}_A^I\right) \cdot \left(\underline{x}_B^I \quad \underline{y}_B^I \quad \underline{z}_B^I\right)^T = \mathbf{D}_A^I \cdot \mathbf{D}_B^{I\,T} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\left(\mathbf{D}_A^B\right)^L = \left(\underline{x}_A^L \quad \underline{y}_A^L \quad \underline{z}_A^L\right) \cdot \left(\underline{x}_B^L \quad \underline{y}_B^L \quad \underline{z}_B^L\right)^T = \mathbf{D}_A^L \cdot \mathbf{D}_B^{L\,T}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$:座標系Lから見た座標系Bから座標系Aへの回転マトリクス

この固定角型回転マトリクスは、基準座標系（カッコの右上添え字）において、カッコ内右上添え字の座標系から右下添え字の座標系への回転を表している。固定角型四元数も同様である。

固定角型回転マトリクスの基準座標系は、「慣性航法のアルゴリズム」に示した式（1.2-13）のように、他の座標系に変換できる。下記に例を示す。

$$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}}=\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\cdot\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}}\cdot\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\ \mathrm{T}}=\begin{pmatrix}0&0&1\\0&1&0\\-1&0&0\end{pmatrix}\cdot\begin{pmatrix}0&-1&0\\0&0&-1\\1&0&0\end{pmatrix}\cdot\begin{pmatrix}0&0&-1\\0&1&0\\1&0&0\end{pmatrix}$$

$$=\begin{pmatrix}1&0&0\\0&0&-1\\0&1&0\end{pmatrix}\cdot\begin{pmatrix}0&0&-1\\0&1&0\\1&0&0\end{pmatrix}=\begin{pmatrix}0&0&-1\\-1&0&0\\0&1&0\end{pmatrix}$$ ：基準座標系を座標系Lから座標系Iへ変換

$$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}}=\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\ \mathrm{T}}\cdot\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}}\cdot\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}=\begin{pmatrix}0&0&-1\\0&1&0\\1&0&0\end{pmatrix}\cdot\begin{pmatrix}0&0&-1\\-1&0&0\\0&1&0\end{pmatrix}\cdot\begin{pmatrix}0&0&1\\0&1&0\\-1&0&0\end{pmatrix}$$

$$=\begin{pmatrix}0&-1&0\\-1&0&0\\0&0&-1\end{pmatrix}\cdot\begin{pmatrix}0&0&1\\0&1&0\\-1&0&0\end{pmatrix}=\begin{pmatrix}0&-1&0\\0&0&-1\\1&0&0\end{pmatrix}$$ ：基準座標系を座標系Iから座標系Lへ変換

次に固定角型四元数を示す。

$$\left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}}=\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\otimes\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}=\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}=\frac{1}{\sqrt{2}}\left\{1+\begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$$

$$\left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}}=\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\otimes\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\ ^{*}}=\frac{1}{2}\left\{1+\begin{pmatrix}-1\\1\\-1\end{pmatrix}\right\}\otimes\frac{1}{\sqrt{2}}\left\{1-\begin{pmatrix}0\\1\\0\end{pmatrix}\right\}=\frac{1}{2\sqrt{2}}\left\{1+\begin{pmatrix}-1\\1\\-1\end{pmatrix}-\begin{pmatrix}0\\1\\0\end{pmatrix}-\begin{pmatrix}1\\0\\-1\end{pmatrix}+1\right\}$$

$$=\frac{1}{\sqrt{2}}\left\{1+\begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}$$

$$\left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}}=\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\otimes\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\ ^{*}}=\frac{1}{2}\left\{-1+\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right\}\otimes\frac{1}{\sqrt{2}}\left\{1-\begin{pmatrix}0\\1\\0\end{pmatrix}\right\}=\frac{1}{2\sqrt{2}}\left\{-1+\begin{pmatrix}-1\\-1\\1\end{pmatrix}+\begin{pmatrix}0\\1\\0\end{pmatrix}-\begin{pmatrix}-1\\0\\-1\end{pmatrix}-1\right\}$$

$$=\frac{1}{\sqrt{2}}\left\{-1+\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$$

$$\left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}}=\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\otimes\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}=\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}=\frac{1}{2}\left\{1+\begin{pmatrix}-1\\1\\-1\end{pmatrix}\right\}$$

$$\left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}}=\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\otimes\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}^{*}}=\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}=\frac{1}{\sqrt{2}}\left\{1+\begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$$

$$\left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}}=\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\otimes\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\ ^{*}}=\frac{1}{\sqrt{2}}\left\{-1+\begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}\otimes\frac{1}{\sqrt{2}}\left\{1-\begin{pmatrix}0\\0\\-1\end{pmatrix}\right\}=\frac{1}{2}\left\{-1+\begin{pmatrix}-1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\-1\end{pmatrix}-\begin{pmatrix}0\\-1\\0\end{pmatrix}\right\}$$

$$=\frac{1}{2}\left\{-1+\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right\}\otimes\left(\frac{1}{2}\left\{1+\begin{pmatrix}-1\\1\\-1\end{pmatrix}\right\}\right)^{*}=\frac{1}{2}\left\{-1+\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right\}\otimes\frac{1}{2}\left\{1-\begin{pmatrix}-1\\1\\-1\end{pmatrix}\right\}$$

$$=\frac{1}{4}\left\{-1+\begin{pmatrix}-1\\-1\\1\end{pmatrix}+\begin{pmatrix}-1\\1\\-1\end{pmatrix}-\begin{pmatrix}0\\-2\\-2\end{pmatrix}-1\right\}=\frac{1}{4}\left\{-2+\begin{pmatrix}-2\\2\\2\end{pmatrix}\right\}$$

$$=\frac{1}{2}\left\{-1+\begin{pmatrix}-1\\1\\1\end{pmatrix}\right\}$$ ：座標系Iから見た座標系Bから座標系Aへの回転を表す四元数

$$\left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}}=\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\otimes\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\ ^{*}}=\frac{1}{\sqrt{2}}\left\{-1+\begin{pmatrix}-1\\0\\0\end{pmatrix}\right\}\otimes\frac{1}{\sqrt{2}}\left\{1-\begin{pmatrix}0\\0\\-1\end{pmatrix}\right\}=\frac{1}{2}\left\{-1+\begin{pmatrix}-1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\-1\end{pmatrix}-\begin{pmatrix}0\\-1\\0\end{pmatrix}\right\}$$

$$= \frac{1}{2}\left\{-1 + \begin{pmatrix}-1\\1\\-1\end{pmatrix}\right\}$$:座標系Lから見た座標系Bから座標系Aへの回転を表す四元数

この四元数を回転マトリクスに変換してみると下記のようになる。

$$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} = \begin{pmatrix} \frac{1}{4}+\frac{1}{4}(1-1-1) & -\frac{1}{2}-\frac{1}{2} & \frac{1}{2}-\frac{1}{2} \\ -\frac{1}{2}+\frac{1}{2} & \frac{1}{4}+\frac{1}{4}(1-1-1) & -\frac{1}{2}-\frac{1}{2} \\ \frac{1}{2}+\frac{1}{2} & -\frac{1}{2}+\frac{1}{2} & \frac{1}{4}+\frac{1}{4}(1-1-1) \end{pmatrix} = \begin{pmatrix}0 & -1 & 0\\ 0 & 0 & -1\\ 1 & 0 & 0\end{pmatrix}$$

固定角型四元数も固定角型回転マトリクスと同様に、基準座標系を他の座標系に変換できる。下記に例を示す。

$$\left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \otimes \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \otimes \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \frac{1}{\sqrt{2}}\left\{1 + \begin{pmatrix}0\\1\\0\end{pmatrix}\right\} \otimes \frac{1}{2}\left\{-1 + \begin{pmatrix}-1\\1\\-1\end{pmatrix}\right\} \otimes \frac{1}{\sqrt{2}}\left\{1 - \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$$

$$= \frac{1}{4}\left\{-1 + \begin{pmatrix}-1\\0\\-1\end{pmatrix} - \begin{pmatrix}-1\\0\\1\end{pmatrix} - 1\right\} \otimes \left\{1 - \begin{pmatrix}0\\1\\0\end{pmatrix}\right\} = \frac{1}{4}\left\{-2 + \begin{pmatrix}0\\0\\-2\end{pmatrix}\right\} \otimes \left\{1 - \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$$

$$= \frac{1}{4}\left\{-2 + \begin{pmatrix}0\\2\\-2\end{pmatrix} + \begin{pmatrix}-2\\0\\0\end{pmatrix} + 0\right\} = \frac{1}{2}\left\{-1 + \begin{pmatrix}-1\\1\\-1\end{pmatrix}\right\}$$

$$\left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \begin{pmatrix}0 & 0 & -1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix} \cdot \begin{pmatrix}0 & 0 & -1\\ -1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix} \cdot \begin{pmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ -1 & 0 & 0\end{pmatrix}$$

$$= \begin{pmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & -1\end{pmatrix} \cdot \begin{pmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ -1 & 0 & 0\end{pmatrix} = \begin{pmatrix}0 & -1 & 0\\ 0 & 0 & -1\\ 1 & 0 & 0\end{pmatrix}$$

2 座標系の軸ベクトル

座標系の軸ベクトルは方向余弦マトリクスの列要素で表される。

$\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{I}}\right)$:座標系Iから見た座標系Lの方向余弦マトリクス

ここでは方向余弦マトリクスと区別して軸ベクトルを次のように表すこととする。

$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \equiv \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{I}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{I}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{I}}^{\mathrm{I}}\right) = [\mathbf{1}]$:座標系Iから見た座標系Iの軸ベクトル

$\mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \equiv \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{L}} \quad \underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{L}} \quad \underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{L}}\right) = [\mathbf{1}]$:座標系Lから見た座標系Lの軸ベクトル

$\mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \equiv \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{I}}\right) = \begin{pmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ -1 & 0 & 0\end{pmatrix}$:座標系Iから見た座標系Lの軸ベクトル

$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \equiv \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{B}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{B}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{B}}^{\mathrm{I}}\right) = \begin{pmatrix}0 & 0 & 1\\ -1 & 0 & 0\\ 0 & -1 & 0\end{pmatrix}$:座標系Iから見た座標系Bの軸ベクトル

$\mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \equiv \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \left(\underline{\mathrm{x}}_{\mathrm{B}}^{\mathrm{L}} \quad \underline{\mathrm{y}}_{\mathrm{B}}^{\mathrm{L}} \quad \underline{\mathrm{z}}_{\mathrm{B}}^{\mathrm{L}}\right) = \begin{pmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$:座標系Lから見た座標系Bの軸ベクトル

$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \equiv \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{A}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{A}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{A}}^{\mathrm{I}}\right) = \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & -1\\ -1 & 0 & 0\end{pmatrix}$:座標系Iから見た座標系Aの軸ベクトル

$\mathbf{x}_{\mathrm{A}}^{\mathrm{L}} \equiv \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \left(\underline{\mathrm{x}}_{\mathrm{A}}^{\mathrm{L}} \quad \underline{\mathrm{y}}_{\mathrm{A}}^{\mathrm{L}} \quad \underline{\mathrm{z}}_{\mathrm{A}}^{\mathrm{L}}\right) = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{pmatrix}$:座標系Lから見た座標系Aの軸ベクトル

3 ベクトル

ここでは回転や座標変換の結果を分かり易くするため各座標軸方向のベクトルのみを用い、演算例を増やしても記述がコンパクトになるよう、演算対象のベクトルを 3 つまとめて次のように表す。

$\mathbf{a}_{\mathrm{I}}^{\mathrm{I}} \equiv \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{I}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{I}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{I}}^{\mathrm{I}}\right) = [\mathbf{1}]$ ：座標系Iから見た座標系Iの軸ベクトル群

$\mathbf{a}_{\mathrm{L}}^{\mathrm{L}} \equiv \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} = \left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{L}} \quad \underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{L}} \quad \underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{L}}\right) = [\mathbf{1}]$ ：座標系Lから見た座標系Lの軸ベクトル群

$\mathbf{a}_{\mathrm{L}}^{\mathrm{I}} \equiv \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{I}}\right) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$ ：座標系Iから見た座標系Lの軸ベクトル群

$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}} \equiv \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{B}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{B}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{B}}^{\mathrm{I}}\right) = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$ ：座標系Iから見た座標系Bの軸ベクトル群

$\mathbf{a}_{\mathrm{B}}^{\mathrm{L}} \equiv \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} = \left(\underline{\mathrm{x}}_{\mathrm{B}}^{\mathrm{L}} \quad \underline{\mathrm{y}}_{\mathrm{B}}^{\mathrm{L}} \quad \underline{\mathrm{z}}_{\mathrm{B}}^{\mathrm{L}}\right) = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ：座標系Lから見た座標系Bの軸ベクトル群

$\mathbf{a}_{\mathrm{A}}^{\mathrm{I}} \equiv \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{A}}^{\mathrm{I}} \quad \underline{\mathrm{y}}_{\mathrm{A}}^{\mathrm{I}} \quad \underline{\mathrm{z}}_{\mathrm{A}}^{\mathrm{I}}\right) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$ ：座標系Iから見た座標系Aの軸ベクトル群

$\mathbf{a}_{\mathrm{A}}^{\mathrm{L}} \equiv \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} = \left(\underline{\mathrm{x}}_{\mathrm{A}}^{\mathrm{L}} \quad \underline{\mathrm{y}}_{\mathrm{A}}^{\mathrm{L}} \quad \underline{\mathrm{z}}_{\mathrm{A}}^{\mathrm{L}}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ ：座標系Lから見た座標系Aの軸ベクトル群

$\mathbf{a}_{\mathrm{I}}^{\mathrm{L}} \equiv \mathbf{x}_{\mathrm{I}}^{\mathrm{L}} = {\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{I}}^{\mathrm{L}} \quad \underline{\mathrm{y}}_{\mathrm{I}}^{\mathrm{L}} \quad \underline{\mathrm{z}}_{\mathrm{I}}^{\mathrm{L}}\right) = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ ：座標系Lから見た座標系Iの軸ベクトル群

$\mathbf{a}_{\mathrm{I}}^{\mathrm{B}} \equiv \mathbf{x}_{\mathrm{I}}^{\mathrm{B}} = {\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{I}}^{\mathrm{B}} \quad \underline{\mathrm{y}}_{\mathrm{I}}^{\mathrm{B}} \quad \underline{\mathrm{z}}_{\mathrm{I}}^{\mathrm{B}}\right) = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$ ：座標系Bから見た座標系Iの軸ベクトル群

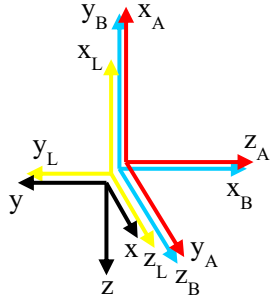
$\mathbf{a}_{\mathrm{L}}^{\mathrm{B}} \equiv \mathbf{x}_{\mathrm{L}}^{\mathrm{B}} = {\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{B}} \quad \underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{B}} \quad \underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{B}}\right) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ：座標系Bから見た座標系Lの軸ベクトル群

$\mathbf{a}_{\mathrm{I}}^{\mathrm{A}} \equiv \mathbf{x}_{\mathrm{I}}^{\mathrm{A}} = {\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{I}}^{\mathrm{I}} = \left(\underline{\mathrm{x}}_{\mathrm{I}}^{\mathrm{A}} \quad \underline{\mathrm{y}}_{\mathrm{I}}^{\mathrm{A}} \quad \underline{\mathrm{z}}_{\mathrm{I}}^{\mathrm{A}}\right) = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$ ：座標系Aから見た座標系Iの軸ベクトル群

$\mathbf{a}_{\mathrm{L}}^{\mathrm{A}} \equiv \mathbf{x}_{\mathrm{L}}^{\mathrm{A}} = {\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}}^{\mathrm{T}} \cdot \mathbf{a}_{\mathrm{L}}^{\mathrm{L}} = \left(\underline{\mathrm{x}}_{\mathrm{L}}^{\mathrm{A}} \quad \underline{\mathrm{y}}_{\mathrm{L}}^{\mathrm{A}} \quad \underline{\mathrm{z}}_{\mathrm{L}}^{\mathrm{A}}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$ ：座標系Aから見た座標系Iの軸ベクトル群

4 回転マトリクスの縮退

回転を目視で容易に確認できるよう、回転マトリクスの縮退形を下記に示す。

$\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = [90°]_2$	$\left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}}$	$\left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = [-90°]_3$	
$\mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = [-90°]_3$	$\left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} = [-90°]_1$	$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = [90°]_1$	
$\mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = [90°]_1$	$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} = [-90°]_3$	$\left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{L}} = [90°]_2 \cdot [90°]_1$	
$\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = [90°]_2 \cdot [-90°]_3$	$\left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}}$	$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{L}} = [90°]_1 \cdot [90°]_2$	
$\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = [90°]_2 \cdot [90°]_1$	$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}}$	$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} = [90°]_1 \cdot [90°]_3$	
$\mathbf{D}_{\mathrm{A}}^{\mathrm{B}} = [90°]_3 \cdot [90°]_1$	$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = [-90°]_3 \cdot [90°]_1$		

5 例題の結果

下記の4種類の体(非可換体)をまとめて、

・方向余弦マトリクス

・固定角型回転マトリクス

・オイラー角型四元数

・固定角型四元数

結果を下記に示す。

表5-1 座標系の回転

表5-2 ベクトルの回転

表5-3 ベクトルの座標変換

なお、体(非可換体)やベクトルを表現する基準の座標系(基準座標系)は、各体(非可換体)やベクトルにおいて、下記の部分で表す。

$\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}$ 、$\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}$:右上添え字の“I”

$\left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}}$ 、$\left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}}$:カッコ外の右上添え字の“I”

$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}}$ 、$\mathbf{a}_{\mathrm{B}}^{\mathrm{I}}$:右上添え字の“I”

表5-1　座標系の回転

No.	方向余弦マトリクスによる回転				固定角型回転マトリクスによる回転			
1	$\mathbf{x}_L^I = \mathbf{x}_I^I \cdot \mathbf{D}_L^I$	◎	$\mathbf{x}_I^I = \mathbf{x}_L^I \cdot \mathbf{D}_L^{I\,T}$	◎	$\mathbf{x}_L^I = \left(\mathbf{D}_L^I\right)^I \cdot \mathbf{x}_I^I$	◎	$\mathbf{x}_I^I = \left(\mathbf{D}_L^I\right)^{I\,T} \cdot \mathbf{x}_L^I$	◎
2	$\mathbf{x}_B^I = \mathbf{x}_L^I \cdot \mathbf{D}_B^L$	◎	$\mathbf{x}_L^I = \mathbf{x}_B^I \cdot \mathbf{D}_B^{L\,T}$	◎	$\mathbf{x}_B^I = \left(\mathbf{D}_B^L\right)^I \cdot \mathbf{x}_L^I$	◎	$\mathbf{x}_L^I = \left(\mathbf{D}_B^L\right)^{I\,T} \cdot \mathbf{x}_B^I$	◎
3	$\mathbf{x}_B^L = \mathbf{x}_L^L \cdot \mathbf{D}_B^L$	◎	$\mathbf{x}_L^L = \mathbf{x}_B^L \cdot \mathbf{D}_B^{L\,T}$	◎	$\mathbf{x}_B^L = \left(\mathbf{D}_B^L\right)^I \cdot \mathbf{x}_L^L$	×	$\mathbf{x}_L^L = \left(\mathbf{D}_B^L\right)^{I\,T} \cdot \mathbf{x}_B^L$	×
4	$\mathbf{x}_A^I = \mathbf{x}_L^I \cdot \mathbf{D}_A^L$	◎	$\mathbf{x}_L^I = \mathbf{x}_A^I \cdot \mathbf{D}_A^{L\,T}$	◎	$\mathbf{x}_A^I = \left(\mathbf{D}_A^L\right)^I \cdot \mathbf{x}_L^I$	◎	$\mathbf{x}_L^I = \left(\mathbf{D}_A^L\right)^{I\,T} \cdot \mathbf{x}_A^I$	◎
5	$\mathbf{x}_A^L = \mathbf{x}_L^L \cdot \mathbf{D}_A^L$	◎	$\mathbf{x}_L^L = \mathbf{x}_A^L \cdot \mathbf{D}_A^{L\,T}$	◎	$\mathbf{x}_A^L = \left(\mathbf{D}_A^L\right)^I \cdot \mathbf{x}_L^L$	×	$\mathbf{x}_L^L = \left(\mathbf{D}_A^L\right)^{I\,T} \cdot \mathbf{x}_A^L$	×
6	$\mathbf{x}_B^I = \mathbf{x}_I^I \cdot \mathbf{D}_B^I$	◎	$\mathbf{x}_I^I = \mathbf{x}_B^I \cdot \mathbf{D}_B^{I\,T}$	◎	$\mathbf{x}_B^I = \left(\mathbf{D}_B^I\right)^I \cdot \mathbf{x}_I^I$	◎	$\mathbf{x}_I^I = \left(\mathbf{D}_B^I\right)^{I\,T} \cdot \mathbf{x}_B^I$	◎
7	$\mathbf{x}_A^I = \mathbf{x}_I^I \cdot \mathbf{D}_A^I$	◎	$\mathbf{x}_I^I = \mathbf{x}_A^I \cdot \mathbf{D}_A^{I\,T}$	◎	$\mathbf{x}_A^I = \left(\mathbf{D}_A^I\right)^I \cdot \mathbf{x}_I^I$	◎	$\mathbf{x}_I^I = \left(\mathbf{D}_A^I\right)^{I\,T} \cdot \mathbf{x}_A^I$	◎
8	$\mathbf{x}_A^I = \mathbf{x}_B^I \cdot \mathbf{D}_A^B$	◎	$\mathbf{x}_B^I = \mathbf{x}_A^I \cdot \mathbf{D}_A^{B\,T}$	◎	$\mathbf{x}_A^I = \left(\mathbf{D}_A^B\right)^I \cdot \mathbf{x}_B^I$	◎	$\mathbf{x}_B^I = \left(\mathbf{D}_A^B\right)^{I\,T} \cdot \mathbf{x}_A^I$	◎
9	$\mathbf{x}_A^L = \mathbf{x}_B^L \cdot \mathbf{D}_A^B$	◎	$\mathbf{x}_B^L = \mathbf{x}_A^L \cdot \mathbf{D}_A^{B\,T}$	◎	$\mathbf{x}_A^L = \left(\mathbf{D}_A^B\right)^L \cdot \mathbf{x}_B^L$	◎	$\mathbf{x}_B^L = \left(\mathbf{D}_A^B\right)^{L\,T} \cdot \mathbf{x}_A^L$	◎

No.	オイラー角型四元数による回転				固定角型四元数による回転			
21	$\mathbf{x}_L^I = \mathbf{Q}_L^I \cdot \mathbf{x}_I^I \cdot \mathbf{Q}_L^{I\,*}$	◎	$\mathbf{x}_I^I = \mathbf{Q}_L^{I\,*} \cdot \mathbf{x}_L^I \cdot \mathbf{Q}_L^I$	◎	$\mathbf{x}_L^I = \left(\mathbf{Q}_L^I\right)^I \cdot \mathbf{x}_I^I \cdot \left(\mathbf{Q}_L^I\right)^{I^*}$	◎	$\mathbf{x}_I^I = \left(\mathbf{Q}_L^I\right)^{I^*} \cdot \mathbf{x}_L^I \cdot \left(\mathbf{Q}_L^I\right)^I$	◎
22	$\mathbf{x}_B^I = \mathbf{Q}_B^L \cdot \mathbf{x}_L^I \cdot \mathbf{Q}_B^{L\,*}$	×	$\mathbf{x}_L^I = \mathbf{Q}_B^{L\,*} \cdot \mathbf{x}_B^I \cdot \mathbf{Q}_B^L$	×	$\mathbf{x}_B^I = \left(\mathbf{Q}_B^L\right)^I \cdot \mathbf{x}_L^I \cdot \left(\mathbf{Q}_B^L\right)^{I^*}$	◎	$\mathbf{x}_L^I = \left(\mathbf{Q}_B^L\right)^{I^*} \cdot \mathbf{x}_B^I \cdot \left(\mathbf{Q}_B^L\right)^I$	◎
23	$\mathbf{x}_B^L = \mathbf{Q}_B^L \cdot \mathbf{x}_L^L \cdot \mathbf{Q}_B^{L\,*}$	◎	$\mathbf{x}_L^L = \mathbf{Q}_B^{L\,*} \cdot \mathbf{x}_B^L \cdot \mathbf{Q}_B^L$	◎	$\mathbf{x}_B^L = \left(\mathbf{Q}_B^L\right)^I \cdot \mathbf{x}_L^L \cdot \left(\mathbf{Q}_B^L\right)^{I^*}$	×	$\mathbf{x}_L^L = \left(\mathbf{Q}_B^L\right)^{I^*} \cdot \mathbf{x}_B^L \cdot \left(\mathbf{Q}_B^L\right)^I$	×
24	$\mathbf{x}_A^I = \mathbf{Q}_A^L \cdot \mathbf{x}_L^I \cdot \mathbf{Q}_A^{L\,*}$	×	$\mathbf{x}_L^I = \mathbf{Q}_A^{L\,*} \cdot \mathbf{x}_A^I \cdot \mathbf{Q}_A^L$	×	$\mathbf{x}_A^I = \left(\mathbf{Q}_A^L\right)^I \cdot \mathbf{x}_L^I \cdot \left(\mathbf{Q}_A^L\right)^{I^*}$	◎	$\mathbf{x}_L^I = \left(\mathbf{Q}_A^L\right)^{I^*} \cdot \mathbf{x}_A^I \cdot \left(\mathbf{Q}_A^L\right)^I$	◎
25	$\mathbf{x}_A^L = \mathbf{Q}_A^L \cdot \mathbf{x}_L^L \cdot \mathbf{Q}_A^{L\,*}$	◎	$\mathbf{x}_L^L = \mathbf{Q}_A^{L\,*} \cdot \mathbf{x}_A^L \cdot \mathbf{Q}_A^L$	◎	$\mathbf{x}_A^L = \left(\mathbf{Q}_A^L\right)^I \cdot \mathbf{x}_L^L \cdot \left(\mathbf{Q}_A^L\right)^{I^*}$	×	$\mathbf{x}_L^L = \left(\mathbf{Q}_A^L\right)^{I^*} \cdot \mathbf{x}_A^L \cdot \left(\mathbf{Q}_A^L\right)^I$	×
26	$\mathbf{x}_B^I = \mathbf{Q}_B^I \cdot \mathbf{x}_I^I \cdot \mathbf{Q}_B^{I\,*}$	◎	$\mathbf{x}_I^I = \mathbf{Q}_B^{I\,*} \cdot \mathbf{x}_B^I \cdot \mathbf{Q}_B^I$	◎	$\mathbf{x}_B^I = \left(\mathbf{Q}_B^I\right)^I \cdot \mathbf{x}_I^I \cdot \left(\mathbf{Q}_B^I\right)^{I^*}$	◎	$\mathbf{x}_I^I = \left(\mathbf{Q}_B^I\right)^{I^*} \cdot \mathbf{x}_B^I \cdot \left(\mathbf{Q}_B^I\right)^I$	◎
27	$\mathbf{x}_A^I = \mathbf{Q}_A^I \cdot \mathbf{x}_I^I \cdot \mathbf{Q}_A^{I\,*}$	◎	$\mathbf{x}_I^I = \mathbf{Q}_A^{I\,*} \cdot \mathbf{x}_A^I \cdot \mathbf{Q}_A^I$	◎	$\mathbf{x}_A^I = \left(\mathbf{Q}_A^I\right)^I \cdot \mathbf{x}_I^I \cdot \left(\mathbf{Q}_A^I\right)^{I^*}$	◎	$\mathbf{x}_I^I = \left(\mathbf{Q}_A^I\right)^{I^*} \cdot \mathbf{x}_A^I \cdot \left(\mathbf{Q}_A^I\right)^I$	◎
28	$\mathbf{x}_A^I = \mathbf{Q}_A^B \cdot \mathbf{x}_B^I \cdot \mathbf{Q}_A^{B\,*}$	×	$\mathbf{x}_B^I = \mathbf{Q}_A^{B\,*} \cdot \mathbf{x}_A^I \cdot \mathbf{Q}_A^B$	×	$\mathbf{x}_A^I = \left(\mathbf{Q}_A^B\right)^I \cdot \mathbf{x}_B^I \cdot \left(\mathbf{Q}_A^B\right)^{I^*}$	◎	$\mathbf{x}_B^I = \left(\mathbf{Q}_A^B\right)^{I^*} \cdot \mathbf{x}_A^I \cdot \left(\mathbf{Q}_A^B\right)^I$	◎
29	$\mathbf{x}_A^L = \mathbf{Q}_A^B \cdot \mathbf{x}_B^L \cdot \mathbf{Q}_A^{B\,*}$	×	$\mathbf{x}_B^L = \mathbf{Q}_A^{B\,*} \cdot \mathbf{x}_A^L \cdot \mathbf{Q}_A^B$	×	$\mathbf{x}_A^L = \left(\mathbf{Q}_A^B\right)^L \cdot \mathbf{x}_B^L \cdot \left(\mathbf{Q}_A^B\right)^{L^*}$	◎	$\mathbf{x}_B^L = \left(\mathbf{Q}_A^B\right)^{L^*} \cdot \mathbf{x}_A^L \cdot \left(\mathbf{Q}_A^B\right)^L$	◎

座標系の回転はオイラー角による方向余弦マトリクスを用いて行うのが一般的である。座標系を表す3軸ベクトルをまとめたものが方向余弦マトリクスであり、軸ベクトルは基準の座標系から見た縦ベクトルで表される。座標系の回転は2つの方向余弦マトリクスの積で表され、方向余弦マトリクスの合成の規則に従う。

$$\mathrm{D}_A^R = \begin{pmatrix} \underline{x}_A^R & \underline{y}_A^R & \underline{z}_A^R \end{pmatrix} = \mathrm{x}_A^R$$:基準の座標系Rから見た座標系Aの3軸ベクトル

$$\mathrm{D}_A^R \cdot \mathrm{D}_G^A = \mathrm{D}_G^R \quad 、 \quad \mathrm{D}_G^{A\,T} = \mathrm{D}_A^G \quad 、 \quad \mathrm{D}_G^G = \mathbf{1}$$

方向余弦マトリクスによる座標系の回転は次のように表される。

$$\begin{pmatrix} \underline{x}_G^R & \underline{y}_G^R & \underline{z}_G^R \end{pmatrix} = \begin{pmatrix} \underline{x}_A^R & \underline{y}_A^R & \underline{z}_A^R \end{pmatrix} \cdot \mathrm{D}_G^A \quad 、 \quad \mathrm{x}_G^R = \mathrm{x}_A^R \cdot \mathrm{D}_G^A$$:座標系の回転

方向余弦マトリクス以外の体（非可換体）も利用できるが、それらを表現する基準の座標系と演算対象の座標軸ベクトルを表現する基準の座標系は一致している必要がある。

(1) 方向余弦マトリクスによる座標系の回転

$$\mathbf{x}_L^I = \mathbf{x}_I^I \cdot \mathbf{D}_L^I = \mathbf{D}_I^I \cdot \mathbf{D}_L^I = \mathbf{D}_L^I = \mathbf{x}_L^I$$

$$\mathbf{x}_B^I = \mathbf{x}_L^I \cdot \mathbf{D}_B^L = \mathbf{D}_L^I \cdot \mathbf{D}_B^L = \mathbf{D}_B^I = \mathbf{x}_B^I$$

$$\mathbf{x}_B^L = \mathbf{x}_L^L \cdot \mathbf{D}_B^L = \mathbf{D}_L^L \cdot \mathbf{D}_B^L = \mathbf{D}_B^L = \mathbf{x}_B^L$$

$$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{L}}$$
$$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{B}\,\mathrm{T}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{A}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{L}}$$

(2) 固定角型回転マトリクスによる座標系の回転

$$\mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{L}}$$
$$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}$$
$$\mathbf{x}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{L}}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{x}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{x}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}}$$
$$\mathbf{x}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}}$$
$$\mathbf{x}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}}\right)^{\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}}\right) \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}}$$

(3) オイラー角型四元数による座標系の回転

四元数によるベクトルの回転は、「慣性航法のアルゴリズム」に示した式（3.4-13）のように、方向余弦マトリクスによるベクトルの回転に置き換えることができ、次のように表すことができる。

$\mathbf{a}_B^L = \mathbf{Q}_B^L \cdot \mathbf{a}_L^L \cdot \mathbf{Q}_B^{L*} = \mathbf{D}_B^L \cdot \mathbf{a}_L^L = \mathbf{a}_B^L$ ：ベクトルの回転（基準座標系が一致している必要がある）

$\mathbf{a}_L^L = \mathbf{Q}_B^{L*} \cdot \mathbf{a}_B^L \cdot \mathbf{Q}_B^L = \mathbf{D}_B^{L\mathrm{T}} \cdot \mathbf{a}_B^L = \mathbf{a}_L^L$ ：ベクトルの回転（基準座標系が一致している必要がある）

この表記を用いて説明する。なお、3つのベクトル群 $\mathbf{x}_B^L = \begin{pmatrix} \underline{x}_B^L & \underline{y}_B^L & \underline{z}_B^L \end{pmatrix}$ を四元数で一気に変換する表現は正しくないが、便宜上、座標系3軸の回転をこのように表現した。

$$\mathbf{x}_L^I = \mathbf{Q}_L^I \cdot \mathbf{x}_I^I \cdot \mathbf{Q}_L^{I*} = \begin{pmatrix} \mathbf{Q}_L^I \cdot \underline{x}_I^I \cdot \mathbf{Q}_L^{I*} & \mathbf{Q}_L^I \cdot \underline{y}_I^I \cdot \mathbf{Q}_L^{I*} & \mathbf{Q}_L^I \cdot \underline{z}_I^I \cdot \mathbf{Q}_L^{I*} \end{pmatrix}$$
$$= \begin{pmatrix} \mathbf{D}_L^I \cdot \underline{x}_I^I & \mathbf{D}_L^I \cdot \underline{y}_I^I & \mathbf{D}_L^I \cdot \underline{z}_I^I \end{pmatrix} = \begin{pmatrix} \underline{x}_L^I & \underline{y}_L^I & \underline{z}_L^I \end{pmatrix} = \mathbf{x}_L^I$$

以下、省略形で表す。

$\mathbf{x}_B^L = \mathbf{Q}_B^L \cdot \mathbf{x}_L^L \cdot \mathbf{Q}_B^{L*} = \mathbf{D}_B^L \cdot \mathbf{x}_L^L = \mathbf{D}_B^L \cdot \mathbf{D}_L^L = \mathbf{D}_B^L = \mathbf{x}_B^L$

$\mathbf{x}_A^L = \mathbf{Q}_A^L \cdot \mathbf{x}_L^L \cdot \mathbf{Q}_A^{L*} = \mathbf{D}_A^L \cdot \mathbf{x}_L^L = \mathbf{D}_A^L \cdot \mathbf{D}_L^L = \mathbf{D}_A^L = \mathbf{x}_A^L$

$\mathbf{x}_B^I = \mathbf{Q}_B^I \cdot \mathbf{x}_I^I \cdot \mathbf{Q}_B^{I*} = \mathbf{D}_B^I \cdot \mathbf{x}_I^I = \mathbf{D}_B^I \cdot \mathbf{D}_I^I = \mathbf{D}_B^I = \mathbf{x}_B^I$

$\mathbf{x}_A^L = \mathbf{Q}_A^L \cdot \mathbf{x}_L^L \cdot \mathbf{Q}_A^{L*} = \mathbf{D}_A^L \cdot \mathbf{x}_L^L = \mathbf{D}_A^L \cdot \mathbf{D}_L^L = \mathbf{D}_A^L = \mathbf{x}_A^L$

$\mathbf{x}_B^I = \mathbf{Q}_B^I \cdot \mathbf{x}_I^I \cdot \mathbf{Q}_B^{I*} = \mathbf{D}_B^I \cdot \mathbf{x}_I^I = \mathbf{D}_B^I \cdot \mathbf{D}_I^I = \mathbf{D}_B^I = \mathbf{x}_B^I$

$\mathbf{x}_A^I = \mathbf{Q}_A^I \cdot \mathbf{x}_I^I \cdot \mathbf{Q}_A^{I*} = \mathbf{D}_A^I \cdot \mathbf{x}_I^I = \mathbf{D}_A^I \cdot \mathbf{D}_I^I = \mathbf{D}_A^I = \mathbf{x}_A^I$

$\mathbf{x}_I^I = \mathbf{Q}_L^{I*} \cdot \mathbf{x}_L^I \cdot \mathbf{Q}_L^I = \mathbf{D}_L^{I\mathrm{T}} \cdot \mathbf{x}_L^I = \mathbf{D}_I^L \cdot \mathbf{D}_L^I = \mathbf{D}_L^L = \mathbf{x}_L^L = \mathbf{x}_I^I$

$\mathbf{x}_L^L = \mathbf{Q}_B^{L*} \cdot \mathbf{x}_B^L \cdot \mathbf{Q}_B^L = \mathbf{D}_B^{L\mathrm{T}} \cdot \mathbf{x}_B^L = \mathbf{D}_B^{L\mathrm{T}} \cdot \mathbf{D}_B^L = \mathbf{D}_L^B \cdot \mathbf{D}_B^L = \mathbf{D}_B^B = \mathbf{x}_B^B = \mathbf{x}_L^L$

$\mathbf{x}_L^L = \mathbf{Q}_A^{L*} \cdot \mathbf{x}_A^L \cdot \mathbf{Q}_A^L = \mathbf{D}_A^{L\mathrm{T}} \cdot \mathbf{x}_A^L = \mathbf{D}_A^{L\mathrm{T}} \cdot \mathbf{D}_A^L = \mathbf{D}_L^A \cdot \mathbf{D}_A^L = \mathbf{D}_A^A = \mathbf{x}_A^A = \mathbf{x}_L^L$

$\mathbf{x}_I^I = \mathbf{Q}_B^{I*} \cdot \mathbf{x}_B^I \cdot \mathbf{Q}_B^I = \mathbf{D}_B^{I\mathrm{T}} \cdot \mathbf{x}_B^I = \mathbf{D}_B^{I\mathrm{T}} \cdot \mathbf{D}_B^I = \mathbf{D}_I^B \cdot \mathbf{D}_B^I = \mathbf{D}_B^B = \mathbf{x}_B^B = \mathbf{x}_I^I$

$\mathbf{x}_I^I = \mathbf{Q}_A^{I*} \cdot \mathbf{x}_A^I \cdot \mathbf{Q}_A^I = \mathbf{D}_A^{I\mathrm{T}} \cdot \mathbf{x}_A^I = \mathbf{D}_A^{I\mathrm{T}} \cdot \mathbf{D}_A^I = \mathbf{D}_I^A \cdot \mathbf{D}_A^I = \mathbf{D}_A^A = \mathbf{x}_A^A = \mathbf{x}_I^I$

基準の座標系が一致していないので変換できない例。

$\mathbf{x}_B^I = \mathbf{Q}_B^L \cdot \mathbf{x}_L^I \cdot \mathbf{Q}_B^{L*} = \mathbf{D}_B^L \cdot \mathbf{x}_L^I = \mathbf{D}_B^L \cdot \mathbf{D}_L^I$

$\mathbf{x}_A^I = \mathbf{Q}_A^L \cdot \mathbf{x}_L^I \cdot \mathbf{Q}_A^{L*} = \mathbf{D}_A^L \cdot \mathbf{x}_L^I = \mathbf{D}_A^L \cdot \mathbf{D}_L^I$

$\mathbf{x}_A^I = \mathbf{Q}_A^B \cdot \mathbf{x}_B^I \cdot \mathbf{Q}_A^{B*} = \mathbf{D}_A^B \cdot \mathbf{x}_B^I = \mathbf{D}_A^B \cdot \mathbf{D}_B^I$

$\mathbf{x}_A^L = \mathbf{Q}_A^B \cdot \mathbf{x}_B^L \cdot \mathbf{Q}_A^{B*} = \mathbf{D}_A^B \cdot \mathbf{x}_B^L = \mathbf{D}_A^B \cdot \mathbf{D}_B^L$

$\mathbf{x}_L^I = \mathbf{Q}_B^{L*} \cdot \mathbf{x}_B^I \cdot \mathbf{Q}_B^L = \mathbf{D}_B^{L\mathrm{T}} \cdot \mathbf{x}_B^I = \mathbf{D}_L^B \cdot \mathbf{D}_B^I$

$\mathbf{x}_L^I = \mathbf{Q}_A^{L*} \cdot \mathbf{x}_A^I \cdot \mathbf{Q}_A^L = \mathbf{D}_A^{L\mathrm{T}} \cdot \mathbf{x}_A^I = \mathbf{D}_L^A \cdot \mathbf{D}_A^I$

$\mathbf{x}_B^I = \mathbf{Q}_A^{B*} \cdot \mathbf{x}_A^I \cdot \mathbf{Q}_A^B = \mathbf{D}_A^{B\mathrm{T}} \cdot \mathbf{x}_A^I = \mathbf{D}_B^A \cdot \mathbf{D}_A^I$

$\mathbf{x}_B^L = \mathbf{Q}_A^{B*} \cdot \mathbf{x}_A^L \cdot \mathbf{Q}_A^B = \mathbf{D}_A^{B\mathrm{T}} \cdot \mathbf{x}_A^L = \mathbf{D}_B^A \cdot \mathbf{D}_A^L$

(4) 固定角型四元数による座標系の回転

$$\mathbf{x}_L^I = \left(\mathbf{Q}_L^I\right)^I \cdot \mathbf{x}_I^I \cdot \left(\mathbf{Q}_L^I\right)^{I*} = \mathbf{Q}_L^I \cdot \mathbf{Q}_I^{I*} \cdot \mathbf{x}_I^I \cdot \left(\mathbf{Q}_L^I \cdot \mathbf{Q}_I^{I*}\right)^* = \mathbf{Q}_L^I \cdot \mathbf{Q}_I^{I*} \cdot \mathbf{x}_I^I \cdot \mathbf{Q}_I^I \cdot \mathbf{Q}_L^{I*}$$
$$= \mathbf{Q}_L^I \cdot \left(\mathbf{Q}_I^{I*} \cdot \mathbf{x}_I^I \cdot \mathbf{Q}_I^I\right) \cdot \mathbf{Q}_L^{I*} = \mathbf{D}_L^I \cdot \left(\mathbf{D}_I^{I\mathrm{T}} \cdot \mathbf{x}_I^I\right) = \mathbf{D}_L^I \cdot \mathbf{D}_I^{I\mathrm{T}} \cdot \mathbf{D}_I^I = \mathbf{D}_L^I = \mathbf{x}_L^I$$

$$\mathbf{x}_B^I = \left(\mathbf{Q}_B^L\right)^I \cdot \mathbf{x}_L^I \cdot \left(\mathbf{Q}_B^L\right)^{I*} = \mathbf{Q}_B^I \cdot \mathbf{Q}_L^{I*} \cdot \mathbf{x}_L^I \cdot \left(\mathbf{Q}_B^I \cdot \mathbf{Q}_L^{I*}\right)^* = \mathbf{Q}_B^I \cdot \mathbf{Q}_L^{I*} \cdot \mathbf{x}_L^I \cdot \mathbf{Q}_L^I \cdot \mathbf{Q}_B^{I*}$$
$$= \mathbf{Q}_B^I \cdot \left(\mathbf{Q}_L^{I*} \cdot \mathbf{x}_L^I \cdot \mathbf{Q}_L^I\right) \cdot \mathbf{Q}_B^{I*} = \mathbf{D}_B^I \cdot \left(\mathbf{D}_L^{I\mathrm{T}} \cdot \mathbf{x}_L^I\right) = \mathbf{D}_B^I \cdot \mathbf{D}_L^{I\mathrm{T}} \cdot \mathbf{D}_L^I = \mathbf{D}_B^I = \mathbf{x}_B^I$$

$$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*}\right)^* = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{A}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}\,*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{A}}^{\mathrm{L}}$$

$$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*}\right)^* \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^* \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{L}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^* \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{L}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*}\right)^* \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{I}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}^*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*}\right)^* \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{I}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{I}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{I}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{B}}^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}^*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}\right)^* \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{I}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{I}}$$

$$\mathbf{x}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}^*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{B}}\right)^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*}\right)^* \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}\,*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}\,*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{L}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{A}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{x}_{\mathrm{B}}^{\mathrm{L}}$$

基準の座標系が一致していないので変換できない例。

$$\mathbf{x}_{\mathrm{B}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^* = \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}}$$

$$\mathbf{x}_{\mathrm{A}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^* = \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{L}} \cdot \mathbf{D}_{\mathrm{L}}^{\mathrm{L}}$$

$$\mathbf{x}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^* \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{B}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{B}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{x}_{\mathrm{B}}^{\mathrm{L}}\right) = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{I}\,\mathrm{T}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}} = \mathbf{D}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{D}_{\mathrm{I}}^{\mathrm{B}} \cdot \mathbf{D}_{\mathrm{B}}^{\mathrm{L}}$$

$$\mathbf{x}_{\mathrm{L}}^{\mathrm{L}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}^*} \cdot \mathbf{x}_{\mathrm{A}}^{\mathrm{L}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{L}}\right)^{\mathrm{I}} = \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}\right)^* \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*} = \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}} \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}\,*}$$
$$= \mathbf{Q}_{\mathrm{L}}^{\mathrm{I}} \cdot \left(\mathbf{Q}_{\mathrm{A}}^{\mathrm{I}\,*} \cdot \mathbf{x}_{\mathrm{L}}^{\mathrm{L}} \cdot \mathbf{Q}_{\mathrm{A}}^{\mathrm{I}}\right) \cdot \mathbf{Q}_{\mathrm{L}}^{\mathrm{